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Increasing quantum communication rates using hyperentangled photonic states

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Quantum communication is based on the generation of quantum states and exploitation of quantum resources for communication protocols. Currently, photons are considered as the optimal carriers of information, because they enable long-distance transition with resilience to decoherence and they are relatively easy to create and detect. Entanglement is a fundamental resource for quantum communication and information processing, and it is of particular importance for quantum repeaters. Hyperentanglement, a state where parties are entangled with two or more degrees of freedom (DoFs) simultaneously, provides an important additional resource because it increases data rates and enhances error resilience. However, in photonics, the channel capacity, i.e., the ultimate throughput, is fundamentally limited when dealing with linear elements. We propose a technique for achieving higher transmission rates for quantum communication by using hyperentangled states, based on multiplexing multiple DoFs on a single photon, transmitting the photon, and eventually demultiplexing the DoFs to different photons at the destination, using Bell state measurements. Following our scheme, one can generate two entangled qubit pairs by sending only a single photon. The proposed transmission scheme lays the groundwork for novel quantum communication protocols with higher transmission rates and refined control over scalable quantum technologies.

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1. INTRODUCTION

Entanglement and quantum correlations between multipartite quantum subsystems have recently attracted significant attention in both fundamental and applied research areas based on quantum mechanics. Multipartite entanglement is the cornerstone of a range of applications, including quantum communications [1,2], quantum computation [3] (also including photonic one-way quantum computing [4–6]), and quantum cryptography [7,8]. There are diverse methods to expand the dimensionality of the Hilbert space. One notable approach involves creating entanglement across multiple degrees of freedom (DoFs) simultaneously, termed "hyperentanglement" [9]. The concept of hyperentanglement [9–11], which entails entangling more than two DoFs simultaneously, plays a pivotal role in facilitating the encoding of information on a large scale. This results in high-capacity quantum communication channels, with photons as the carriers of information, facilitating the means to manipulate entanglement. Since long-range communication is nowadays based on single-mode optical fibers, the prospect of encoding larger amounts of quantum information on single photons (rather than on pairs of photons or clusters) makes them attractive, because this can increase the channel capacity. The volume of information carried by single photons can be potentially enormous.

In the realm of photon-based quantum information, various DoFs of light are leveraged to implement qubits on any physical states viewed as two-level states. These include, for example, spin angular momentum (SAM), spatial distribution of the light beam [e.g., orbital angular momentum (OAM) encoding] [12–14], path encoding (propagation direction or k-vector) [15], and time [16] (time-bin and time-frequency encoding) [11,17–19]. While standard entanglement typically involves pairs of photons with DoFs, such as polarization or path, in principle any binary quantum alternative can serve as a qubit, including internal DoFs [12]. Consequently, a single quantum particle may inherently possess multiple DoFs, allowing the encoding of several qubits on a single particle. In this context, these DoFs are entangled at the same time but are not entangled between themselves. Hyperentangled states are often expressed as product states of entangled states, where each subsystem is entangled in a DoF.

Hyperentanglement offers significant advantages in quantum communication protocols, e.g., secure superdense coding [20] and cryptography [21–23]. For example, if the DoFs of a hyperentangled pair of particles can be considered as a qubit, then the state of the hyperentangled pair is a tensor product of Bell states in each of the $n \otimes n$ variables (each representing an effective qubit encoded in the DoF). Thus, there are 4^n entangled states overall. This construction upgrades the quantum communication channel, as it doubles the information capacity with more qubits multiplexed in each pair of particles. In this vein,

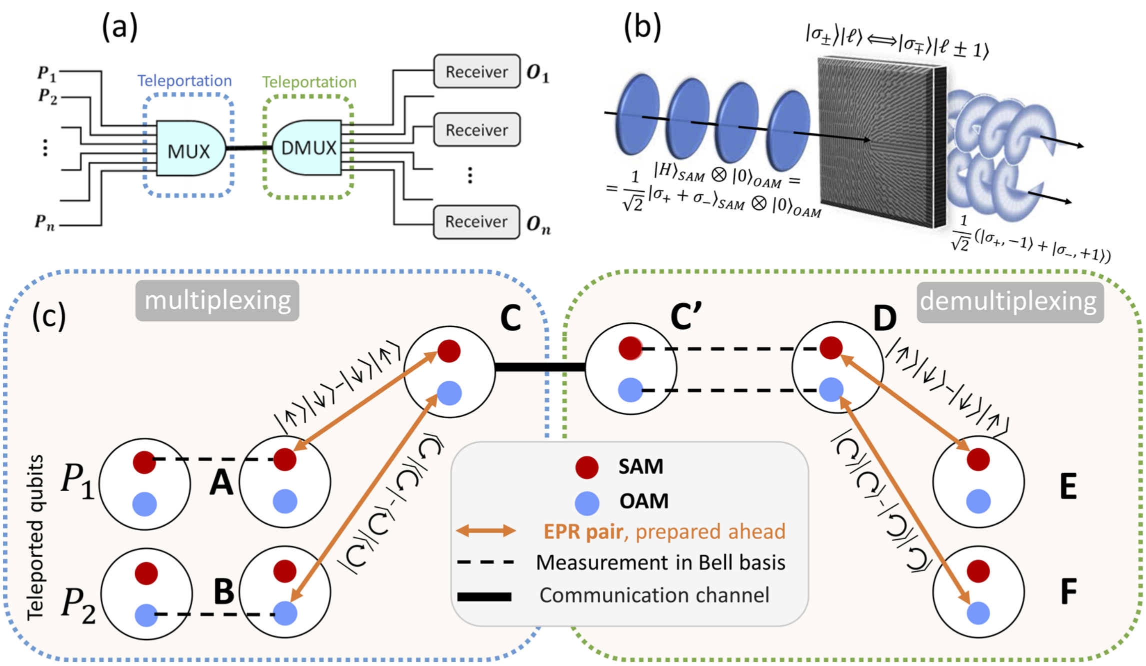

Fig. 1. (a) General scheme for the increasing communication rate with N DoFs. MUX – multiplexing; blue dashed line highlights the multiplexing process (using Bell state measurements) detailed in (c). DMUX – demultiplexing; green dashed line encompasses the process described in (c). (b) Example for encoding a qubit on a single photon. The metasurface that entangles between SAM and OAM of a photon. The photon carries zero OAM before passing through the metasurface, after which it exits as a superposition of two circular polarizations, with the corresponding vortex phase fronts opposite to one another [14]. The metasurface can be used to prepare the initial state for our protocol. (c) Scheme for multiplexing, transmitting, and demultiplexing hyperentanglement using Bell state measurements. The multiplexing part (demultiplexing) is performed by Bell state measurements of the SAM and OAM DoFs from separate photons (single photon) to a single photon (separate photons) and is labeled by the dashed blue (green) line.

expanding the hyperentanglement to even more DoFs offers higher information capacity, i.e., a large Hilbert space can be achieved through entanglement in more than one DoF, and the DoF can also be expanded to more than two dimensions (known as high-dimensional entanglement), such as OAM DoF [12] or frequency [11].

In recent years, multipartite entanglement has seen a growing focus [24–28], due to the ever-increasing demand for high-capacity channels, where hyperentangled photonic states provide a high-capacity link for sending information by more efficient encoding on each physical photon. However, coding information on single photons also involves some limitations, since this means that the nonclassical correlations manifesting the entanglement are always local. Thus, it appears that entangling multiple qubits on a single photon precludes the use of one of the main advantages of entanglement: the ability to measure the correlations between the qubits in separate locations.

Here, we show how to overcome the limitations of encoding multiple qubits on a single photon. We present a scheme for the transmission of qubits at a higher rate by multiplexing N qubits on a single photon using Bell state measurements [29], transmitting a string of single photons to a distant location, and eventually—at the destination—demultiplexing the quantum information to N photons, each carrying a single qubit [Fig. 1(a)]. This increases the quantum capacity of the channel while still allowing the information to be processed in parallel at the destination, exploiting the nonlocality of quantum information processing.

2. MULTIPLEXING–DEMULTIPLEXING SCHEME

Our method allows the encoding of a stream of single photons, each acting as an "information carrier" photon, with comprehensive information encompassing all the other photons within the state. This stands in contrast to conventional hyperentangled states, where multiple photons are entangled across various DoFs without a primary photon containing the entire quantum state information. Moreover, our technique is general and can be applied to any set of DoFs without restriction to a specific implementation. As a concrete example, we use the DoFs of SAM and OAM, where we perform Bell state measurements of the SAM and OAM DoFs from two separate photons and code the quantum information on a single photon [30]. The technique has two parts [Fig. 1(c)]: a multiplexing part, where we measure in Bell basis the SAM of photon P_1 with photon A and the OAM of photon P_2 with photon B, followed by a demultiplexing part, where we measure (also in Bell basis) the SAM and OAM of photon C and photon D. The encoding protocol comprises of photon C (the information carrier) and photons A and B, which are entangled (separately) in their SAM and OAM with photon C. The decoding protocol comprises photons D, E, and F. Photon C, the information carrier, is transmitted through the quantum channel until it reaches the receiver, where it is presented as photon C' and the encoded information can now be demultiplexed.

For the encoding and decoding of qubits, we employ two pairs of hyperentangled 3-photon states. The first state comprises photons A, B, and C, while the second state involves photons D, E, and F. The generation of these two pairs can be achieved using a metasurface [Fig. 1(b)]: a mask imprinting a different wavefunction to each polarization of the electromagnetic field. These devices were recently used to imprint entanglement between the SAM and the OAM of a photon [14].

To design a metasurface that entangles the photon SAM with OAM, the nanoantenna orientations are designed in such a way that the mask adds or subtracts $\Delta\ell = 1$ (one quanta of OAM), depending on the sign of the spin, and performs a spin-flip $|\sigma_+\rangle \leftrightarrow |\sigma_-\rangle$. Such a metasurface [14], presented in Fig. 1(b), performs the unitary transformation $|\sigma_\pm\rangle|\ell\rangle \Leftrightarrow |\sigma_\mp\rangle|\ell \pm \Delta\ell\rangle$, where $\sigma_\pm$ represents the spin states of the photon (right- and left-handed circular polarizations), and ℓ represents the OAM of the photon.

To initiate the multiplexing part, our transmitter utilizes photons P_1 and P_2. A Bell measurement is conducted on the SAM of photons P_1 and A, as well as the OAM of photons P_2 and B [Fig. 1(c)]. To implement the multiplexing protocol of both SAM and OAM from photons P_1 and P_2 to photon C, the transmitter uses the 3-photon state consisting of photons A, B, and C. The collective state of the three photons can be described as follows:

$$|\sigma_+, \mathrm{L_A}\rangle_A \otimes |S_B, -1\rangle_B \otimes |\sigma_-, +1\rangle_C + |\sigma_-, \mathrm{L_A}\rangle_A \otimes |S_B, +1\rangle_B \otimes |\sigma_+, -1\rangle_C, \quad (1)$$

where ± 1 represent one quanta of OAM, S_B is the SAM DoF of photon B, and L_A is the OAM of photon A (which is not utilized for information storage on these DoFs). Equation (1) describes a state with photon A and photon C entangled in their SAM, while photons B and C are entangled in their OAM. Their combination yields a set of 16 hyperentangled Bell states. Notably, the state in Eq. (1) is hyperentangled, possessing entanglement in each quantum variable concurrently and independently, while there is a primary photon (photon C) that encapsulates the information stored in the two other photons by being entangled simultaneously in two different DoFs with both of them. It is important to highlight that this hyperentangled state can autonomously serve as a valuable resource for a variety of quantum communication protocols.

After conducting the two-particle joint measurement involving photon A with photon P_1, and photon B with photon P_2, the transmitter projects each of these pairs onto the 16-basis of orthogonal and complete hyperentangled Bell states, discriminating one of them in each measurement. The measurements of photons A with P_1, and photons B with P_2, are encoded as four-bit classical information. Following this, appropriate Pauli operations are applied to the SAM and OAM of photon C, enabling the perfect reconstruction of the initial SAM of photon P_1 and the initial OAM of photon P_2. A detailed description is presented in the Supplement 1, section B. This scheme is presented by the blue dashed line in Fig. 1(c). Such a mechanism is an effective way of multiplexing quantum information on a single photon. The single photon can now be transmitted to a distant destination, where the information can be demultiplexed into multiple photons, facilitating nonlocal quantum operations. The transmission then occurs by transmitting a single photon carrying two qubits, which doubles the quantum transmission rate.

To demultiplex the quantum information at the receiver, the local process is repeated, separating the DoFs from photon C' to photons E and F, demultiplexing the hyperentangled state. To demultiplex the SAM and OAM DoFs in photon C', the receiver prepares ahead a hyperentangled state, analogous to the multiplexing part, consisting of three photons—D, E, and F. This

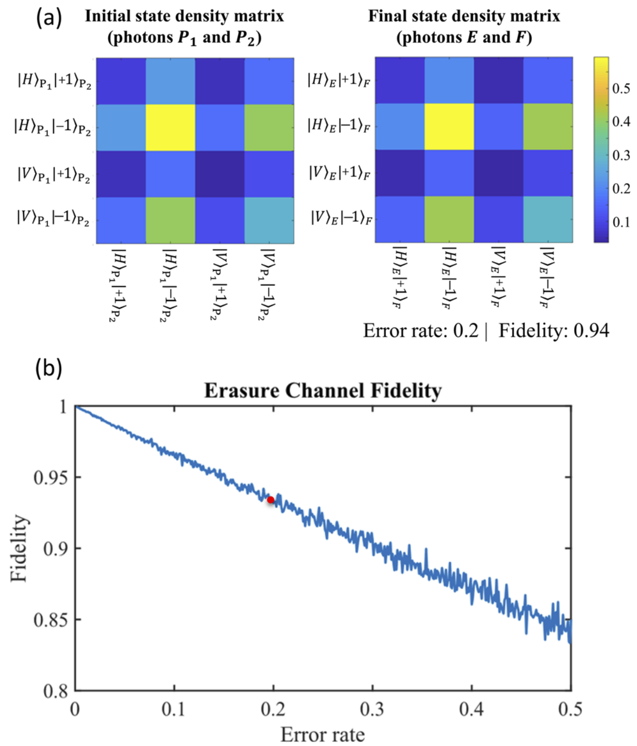

Fig. 2. (a) Density matrices of the initial (before the multiplexing stage) and final (after the demultiplexing stage) states, with error rate (i.e., the chances of a photon to get lost) of 0.2 [red dot in (b)]. The initial state is randomly generated. (b) Calculated fidelity as a function of error rate. Each point represents the average of 70 randomized states with the same error rate.

state comprises two Bell states: photons D and E, which are entangled in their SAM, and another Bell pair of photons D and F, which are entangled in their OAM. Subsequently, qubits encoded in photons C and D are measured in Bell basis, with one measurement for the SAM of both photons and the second for the OAM of both photons. By interpreting and decoding the results, the decoder can apply unitary gates on photons E and F (entangled with photon D in the SAM and OAM, respectively) to perfectly reconstruct the SAM and OAM qubits. Thus, using only linear elements, the decoder has locally demultiplexed two qubits encoded on a single photon to two distinct photons. In a fashion similar to the multiplexing process, our decoder separates the DoFs from the information carrier photon and demultiplexes the qubits to multiple photons (each DoF on a different photon). To be able to perform deterministic multiplexing and demultiplexing, one should be able to perform a complete Bell state measurement, which cannot be performed using "standard" Einstein–Podolsky–Rosen (EPR) pairs and linear optics only [31,32]. However, using hyperentanglement, one can actually isolate every individual Bell measurement (out of four possible outcomes) with a 100% certainty [33–39].

3. RESULTS AND DISCUSSION

At this point it is essential to calculate the quantum capacity of the channel. In communication theory, the term "channel use" refers to a single transmission over the communication channel [40]. Thus, in our context, we examine the transmission of a single photon through the system, and the quantum capacity is defined as the best ratio of information qubits per transmission, i.e., per single photon. In the following we simulate the process in the presence of noise to assess its performance in terms of quantum capacity and fidelity. The results are shown in Fig. 2. The reconstructed state is presented in Fig. 2(a), and the calculated fidelity v. error rate in Fig. 2(b). The Fidelity is defined as $F = Tr(\rho_f |\psi_i\rangle\langle\psi_i|) = Tr(\rho_f \rho_i)$, i.e. – the overlap of the ideal teleported state $|\psi\rangle$ and the measured density matrix ρ_f at the side of the receiver. We simulate (using [41]) the protocol by sending two qubits of information using the transmission of a single photon through a noisy quantum channel. The most common noise in quantum optical communication protocols is the loss of photons. Therefore, to simulate the noise, we assume an erasure channel where a transmitter sends a qubit, and the receiver either receives the qubit correctly or loses the qubit with some probability. With this in mind, we introduce photon loss noise after each operation of the multiplexing–demultiplexing protocol presented in Fig. 1(c). Finally, to evaluate the performance of our system in the presence of noise, we measure the fidelity between the initial and the reconstructed states, while increasing the erasure rate (the probability that the photon is lost).

Let us first examine the standard case of an erasure channel in a system with a single qubit per transmitted photon. The quantum capacity is

$$C_Q = 1 - 2\varepsilon \text{ for } 0 \leq \varepsilon \leq \frac{1}{2}, \quad (2)$$

where ε is the erasure rate. Otherwise, if $\frac{1}{2} \leq \varepsilon \leq 1$, $C_Q = 0$, because more than half the photons are lost to the environment. In contradistinction, in our system the quantum capacity is doubled, as derived in Eq. (7), that is

$$C_Q = 2(1 - 2\varepsilon) \text{ for } 0 \leq \varepsilon \leq \frac{1}{2}. \quad (3)$$

Thus, our protocol improves the quantum capacity, i.e., the ratio of information qubits per transmission, by increasing the number of DoFs available for representing the qubits and multiplexing them on a single photon.

Generally speaking, in network information processing tasks, one may distinguish between local and nonlocal resources. For example, the transmission of a qubit between remote locations is a nonlocal resource. On the other hand, the application of quantum gates and (local) Bell measurements, performed at the transmitter and the receiver separately, are considered to be local resources. Entanglement can be either local or nonlocal. A Bell state between two qubits in the transmitters is a local resource. However, if one of the qubits is sent to the receiver, then the entanglement resource becomes nonlocal. In quantum computing, the focus is often on the local resources of complexity. On the other hand, in communication theory, the focus is on the conversion of nonlocal resources, while the local state preparation, measurements, and operation costs are ignored [42]. Our system is intended for quantum communication, hence we analyze the communication performance and channel capacity. Consequently, we consider the conversion of nonlocal resources, while the complexity of the local multiplexing–demultiplexing procedures is not included in our analysis. An example of an application that requires the use of nonlocal resources is distributed quantum computation [43,44]: it refers to the idea of distributing quantum computing tasks across multiple quantum processors or quantum computers, connected in a network. This approach is motivated by the challenges associated with building large-scale, fault-tolerant quantum computers, as well as the potential advantages of leveraging distributed resources.

Nonetheless, in the pursuit of generating those local resources of hyperentangled states (which enable the encoding of more qubits per photons), various experimental challenges arise. One such challenge involves efficiency detecting all photons. A potential solution to overcome this challenge is the utilization of a photon number resolving detector (PNRD) [45], a specialized device designed for accurately determining the exact number of photons in a given light pulse. Unlike conventional photodetectors, which can only provide information about the presence of photons but not their exact number, PNRDs offer the capability to resolve the photon number with a high degree of precision. These detectors prove instrumental in post-selection processes used for entangling photons, and thereby can enhance the fidelity of our resource hyperentangled states compared to standard single-photon detectors. Another method is to use quantum non-demolition measurement (QND), whereby a single photon can be observed without destroying it and keeping its quantum information intact [46,47].

Another challenge lies in performing a complete Bell state measurement across multiple DoFs (that is, hyperentangled Bell state measurement). However, recent advancements suggest that this technological hurdle can be solved, demonstrating teleportation of multiple DoFs on a single photon within fidelity suppressing classical limits [30], as well as teleportation in high dimensions [48,49] and between different DoFs [50].

These technological enhancements, coupled with recent experimental achievements in the teleportation of multiple DoFs, underscore that, despite encountering some experimental challenges and photon losses, our proposed scheme has the potential to significantly boost data rates per photon suppressing the throughput achievable through the standard encoding of a single qubit per photon.

In conclusion, distinguishing between the theoretical calculation of the quantum channel capacity and its associated communication resource requirements, and the practical experimental challenges inherent to the local resources required for the protocol, is important. Specifically, the practical challenges in our protocol pertain to the implementation of local hyperentangled states and the performance of hyperentangled Bell state measurements at both ends—at the transmitter and at the receiver.

In the subsequent discussion, we provide a detailed exploration of the performance of the improved quantum channel within our setup.

3.1. Enhanced Quantum Capacity

The quantum capacity is the ratio of information qubits per physical photon transmission. Every quantum channel $\mathcal{N}_{A\to B}$ has a Stinespring representation $\mathcal{N}_{A\to B}(\rho) = \mathrm{Tr}_{\mathrm{E}}(V\rho V^\dagger)$ for every input density operator ρ on the Hilbert space $\mathcal{H}_A$, where the operator $V : \mathcal{H}_A \to \mathcal{H}_B \otimes \mathcal{H}_E$ satisfies $V^\dagger V = \mathbb{I}$. We refer to the systems A, B, and E as belonging to Alice, Bob, and the environment, respectively.

Given a joint state $|\phi_{A\bar{A}}\rangle$, where $\bar{A}$ is an ancilla, denote the corresponding joint state of Bob, the environment, and the ancilla by $|\omega_{BE\bar{A}}\rangle \equiv (V \otimes \mathbb{I})|\phi_{A\bar{A}}\rangle$. The coherent information serves as a measure for the rate of quantum information that can be reliably transmitted through the quantum channel from Alice to Bob. The coherent information $I(\bar{A}\rangle B)_\omega$, from $\bar{A}$ (Alice) to B (Bob), is defined as

$$I(\bar{A}\rangle B)_{\omega} \equiv S(\omega_B) - S(\omega_{BA_1}) = S(\omega_B) - S(\omega_E), \quad (4)$$

where ω_B, ω_{BA_1}, and ω_E are the reduced density operators of the respective systems, as $S(\omega) = -\mathrm{Tr}(\omega \log(\omega))$ denotes the von Neumann entropy with respect to the density operator ω. The coherent information of the channel is defined as $I(\mathcal{N}) \equiv \max_{|\phi_{A\bar{A}}\rangle} I(\bar{A}\rangle B)_{\omega}$, where we optimize over all possible input states $|\phi_{A\bar{A}_1}\rangle$.

Based on the Lloyd–Shor–Devetak result [51–53], a.k.a. the LSD theorem, the quantum capacity $C_Q(\mathcal{N})$ of a given channel $\mathcal{N}_{A\to B}$ is given by the following formula:

$$C_Q(\mathcal{N}) \equiv \lim_{n\to\infty} \frac{1}{n} I(\mathcal{N}^{\otimes n}), \quad (5)$$

where $\mathcal{N}$ is the quantum channel and n is the number of transmitted photons through the quantum channel. In principle, to compute the quantum capacity for a general quantum channel, we need to compute the coherent information for the n-fold product channel $\mathcal{N}^{\otimes n}$, normalize by n, and then take n to infinity. This could be difficult to compute. However, for the class of degradable channels, the characterization reduces to a much simpler formula: $C_Q(\mathcal{N}) \equiv I(\mathcal{N})$. Intuitively, a quantum channel is degradable when the channel from Alice to the environment is noisier than the channel from Alice to Bob.

In the standard case of an erasure channel, the quantum channel capacity is given by $C_Q = 1 - 2\varepsilon$ for $0 \le \varepsilon \le \frac{1}{2}$, where ε is the erasure rate. Otherwise, if $\frac{1}{2} \le \varepsilon \le 1$, the quantum capacity is zero. Now, we calculate the quantum channel capacity of our case, where we transmit two qubits over a single transmission (a single photon contains two qubits).

To show achievability of the quantum channel capacity of our protocol, we calculate the overall coherent information in our model, from $\boldsymbol{A_1A_2}$ to $\boldsymbol{B_1B_2}$, according to Eq. (4). Suppose that we encode the input qubits such that $|\phi_{A_1\bar{A}_1}\rangle \otimes |\phi_{A_2\bar{A}_2}\rangle$ is a product of EPR pairs. Then

$$I(\bar{A}_1\bar{A}_2\rangle B_1B_2)_{\rho} = S(B_1B_2Z)_{\rho} - S(\bar{A}_1\bar{A}_2B_1B_2Z)_{\rho}, \quad (6)$$

where Z is a classical indicator that takes the value 1 if an erasure occurred, and otherwise 0 if the photon was not erased, and we describe a classical-quantum state as $\rho_{ZB} = \sum_Z p_z(z)|z\rangle\langle z| \otimes \rho_B^z$. The output B_i can be viewed as a qutrit, $\mathcal{H}_B = \mathbb{C}^{\{0,1,e\}}$, since the erasure state is orthogonal to the qubit's states. Now,

$$\begin{aligned} I(\bar{A}_1\bar{A}_2\rangle B_1B_2)_{\rho} &= S(B_1B_2|Z) - S(A_1A_2B_1B_2|Z) \\ &= (1-p)S\left(\frac{\mathbb{I}_{B_1}}{2} \otimes \frac{\mathbb{I}_{B_2}}{2}\right) + p \cdot S(|e\rangle\langle e| \otimes |e\rangle\langle e|) \\ &\quad - (1-p)S(\phi_{\bar{A}_1\mathrm{B}_1} \otimes \phi_{\bar{A}_2\mathrm{B}_2}) \\ &\quad - p \cdot S\left(\frac{\mathbb{I}_{B_1}}{2} \otimes \frac{\mathbb{I}_{B_2}}{2} \otimes |e\rangle\langle e| \otimes |e\rangle\langle e|\right), \end{aligned} \quad (7)$$

since the conditional entropy satisfies $S(B|Z)_{\rho} = \sum_Z p_z(z)S(\rho_B^z)$ and the indicator Z is distributed according to Bernoulli (p). Note that if erasure occurred, then the output state is $|e\rangle\langle e| \otimes |e\rangle\langle e|$. Otherwise, if erasure has not occurred, then the output state is identical to the reduced input state, $\frac{\mathbb{I}}{2} \otimes \frac{\mathbb{I}}{2}$.

Now, for product states, $\rho_{AB} = \rho_A \otimes \rho_B$, we have $S(AB)_{\rho} = S(A)_{\rho} + S(B)_{\rho}$. Thus, Eq. (7) reduces to

$$I(\bar{A}_1\bar{A}_2\rangle B_1B_2)_{\rho} = \mathbf{2 - 4}\boldsymbol{p}, \quad (8)$$

which means that the coherent information, which in our case is the quantum capacity, is doubled compared to the standard encoding of a single DoF on a photon. A detailed description of this calculation is given in the Supplement 1.

As a final step, we show that the quantum capacity is increased while multiplexing more qubits on a single photon. One may notice that in the general case, where we multiplex n DoFs, the quantum capacity will increase by a factor of n, following the same derivation:

$$\begin{aligned} &I(\bar{A}_1\bar{A}_2\ldots\bar{A}_n\rangle B_1B\ldots B_n)_{\rho} \\ &\quad = S(B_1B_2\ldots B_n)_{\rho} - S(\bar{A}_1\bar{A}_2\ldots\bar{A}_nB_1B_2\ldots B_n)_{\rho} \\ &\quad = S(B_1B_2\ldots B_n, Z)_{\rho} - S(\bar{A}_1\bar{A}_2\ldots\bar{A}_n, B_1B_2\ldots B_n, Z)_{\rho} \\ &\quad = S(B_1B_2\ldots B_n|Z) - S(\bar{A}_1\bar{A}_2\ldots\bar{A}_n, B_1B_2\ldots B_n|Z) \\ &\quad = (1-p)S\left(\frac{\mathbb{I}_{\bar{A}_1}}{2} \otimes \frac{\mathbb{I}_{\bar{A}_2}}{2} \ldots \otimes \frac{\mathbb{I}_{\bar{A}_n}}{2}\right) \\ &\qquad + p \cdot S(|e\rangle\langle e| \otimes |e\rangle\langle e| \ldots \otimes |e\rangle\langle e|) \\ &\qquad - (1-p)S(\phi_{\bar{A}_1B_1} \otimes \phi_{\bar{A}_2B_2} \ldots \otimes \phi_{\bar{A}_nB_n}) - p \\ &\qquad \cdot \mathrm{S}\left(\frac{\mathbb{I}_{\bar{A}_1}}{2} \otimes \frac{\mathbb{I}_{\bar{A}_2}}{2} \ldots \otimes \frac{\mathbb{I}_{\bar{A}_n}}{2} \otimes |e\rangle\langle e| \otimes |e\rangle\langle e| \ldots \otimes |e\rangle\langle e|\right) \\ &\quad = (\mathbf{1 - 2}\boldsymbol{p})\boldsymbol{n}. \end{aligned} \quad (9)$$

This means, that our technique of multiplexing qubits on a single photon—instead of transmitting several photons while each is encoded with a single qubit—has a higher quantum channel capacity. In fact, we can increase the quantum channel capacity more than twice compared to the standard case if we multiplex more than two DoFs.

Thus far, we have described the process in detail for two DoFs. However, this protocol of multiplexing qubits on a single photon, transmitting it, and eventually demultiplexing into multiple photons while recovering the full quantum information, can be scaled up by increasing the size of the Hilbert space of the transmitted qubits. We distinguish between two strategies to increase the Hilbert space: (1) adding k DoFs as two-level systems (i.e., adding more qubits); and (2) adding a DoF as an n-level system, where $n = 2^k$ (in other words, rather than expanding the number of DoFs, the approach involves utilizing some of the existing DoFs as qudits). From a purely mathematical perspective, the two options follow the same trend. Both actions enlarge the Hilbert space and enable the multiplexing of a greater amount of information on a single photon. However, the latter strategy, working with a DoF considered as an n-level system, leads to a linear increase in space dimension. In contrast, the first approach of introducing several DoFs, each treated as a qubit, results in an exponential increase in the Hilbert space. This is due to the Hilbert space being defined as $\{2^N \otimes 2^N \otimes \ldots \otimes 2^N\}$. Consequently, hyperentanglement, i.e., encoding simultaneously on several DoFs, scales more rapidly in the dimensionality of the Hilbert space, facilitating the storage of additional quantum information on each qubit.

Ideally, one can increase the Hilbert space by adding an indefinite number of DoFs and encoding all of them on a single photon, thereby increasing the quantum channel capacity by k or $n = 2^k$, respectively. However, there are practical limits to the number of different DoFs that can be carried by a single photon. For example, the number of detectors required for measuring is equal to the number of encoded qubits, not to the number of the physical photons. This sets a technological limit on the number of DoFs that can be encoded on a single photon. Additionally, expanding with more DoFs restricts the size of the Hilbert space suitable for Bell tests, as required by the quantum communication procedure.

Another method for increasing the dimensionality in the second strategy is by expanding the number of modes in the OAM DoF to more than two (i.e., encode a qubit in this DoF). For example, the metasurface can be designed to endow each polarization with multiple states of OAM, resulting in multiplexing of high-dimensional qubits on the information carrier photon. Another example, perhaps even more important, is employing frequency as a DoF, where the number of possibilities can be enormous (the number of peaks in a frequency comb) [17]. Here, dispersive effects in the fiber may restrict the number of possibilities, but these are not conceptual, and some might be accounted for in advance in the coding scheme.

As a case study, we can use two DoFs, such as frequency and SAM. The SAM DoF is a two-level system, and the frequency DoF can be extended to be N-dimensional. These two DoFs are orthogonal and can be encoded together on a single physical photon (i.e., the combined product channel). When encoding the DoFs together, the qubits transmission rate for general noise models can be larger than the sum of the quantum capacities for each one separately. This unique feature is known as the superadditivity of the quantum capacity [54]. In addition, in some cases we can even achieve superactivation, where the quantum channel capacity of each channel (for each DoF) is zero, but the quantum channel capacity of the combined product channel takes a positive value [54]. This feature is unique to quantum communication and does not occur in classical communication. For detailed discussion about the two schemes for increasing the Hilbert space of our qubits (thus increasing the rate of communication) and about superactivation, see the Supplement 1.

One may also consider the task of sending classical information using the quantum channel, as in the simple setting of superdense coding [20]. The ultimate rate of classical information per quantum transmission is called the classical capacity of the quantum channel. Whether superadditivity applies to the classical capacity of a quantum product channel is yet an open problem [2,55].

Thus far, we have presented a technique that increases the transmission rate. However, since the quantum capacity also determines the rate of generating entangled pairs in remote locations, this protocol is also valuable for entanglement generation for quantum communication tasks, such as quantum key distribution and superdense coding [20]. In the latter, we can use a Bell pair to transmit two classical bits over a single channel (a single qubit), doubling the transmission rate. In our setting, we can multiply the transmission rate by a factor of four using the doubled generation of photon pairs. Figure 3 describes a similar setting for another purpose—generation of nonlocal entanglement between a transmitter and a receiver far apart from each other—and here too our method facilitates entanglement generation at twice the rate, i.e., by sending only a single photon, we generate two nonlocal pairs of entangled photons at the destination. Our transmitter prepares ahead two entangled pairs: the SAM of photons P_3 and P_1, and the OAM of photons P_2 and P_4. Then, our transmitter performs a Bell state measurement of the SAM of photon P_1 with photon A, and the OAM of photon P_2 with photon B (as in Fig. 3), thus creating entanglement between the SAM of photon C with the SAM of photon P_3, and the OAM of photon C with the OAM of photon P_4. After the demultiplexing part, we end up with two separate pairs of entangled photons: photon E is entangled with photon P_3 (in SAM DoF), and photon F is entangled with photon P_4 (in OAM DoF).

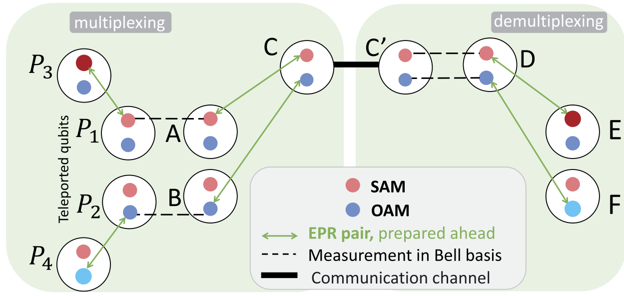

Fig. 3. Scheme for generating entangled photon pairs at a double rate. We start with initial states of the SAM of photon $\boldsymbol{P}_1$ entangled with the SAM of photon $\boldsymbol{P}_3$, and the OAM of photon $\boldsymbol{P}_2$ entangled with the OAM of photon $\boldsymbol{P}_4$. The green line represents entanglement between two DoFs, the black dashed line represents a measurement in Bell basis, and the solid black line represents the communication channel. At the output, we obtain the SAM of photon E entangled with the SAM of photon $\boldsymbol{P}_3$, and the OAM of photon F entangled with OAM of photon $\boldsymbol{P}_4$.

This feature of generating entangled pairs of photons at twice the rate can be important in entanglement-assisted communication, increasing the channel capacity—specifically the entanglement-assisted classical capacity—which is the highest rate at which classical information can be transmitted from a sender to a receiver sharing an unlimited amount of noiseless entanglement. Our protocol not only increases the transmission rate of a quantum communication channel, but also increases the rate of generating entangled pairs of qubits in remote locations.

4. CONCLUSIONS

To conclude, we proposed a protocol to enhance the transmission rate of qubits. This is achieved by multiplexing N qubits onto a single photon through Bell state measurements, sending the single photon, and then, upon reaching the destination, demultiplexing the quantum information onto N different photons, which can be used for parallel processing of the quantum information. This system preserves the correlations between the different DoFs, despite being all carried by a single photon. Our protocol can also be valuable for entanglement generation, i.e., the protocol can generate two entangled qubit pairs by sending only a single photon. This can be useful for various quantum communication protocols, such as quantum key distribution, superdense coding, etc.

As technology evolves, enhancing quantum capacities, exemplified by our approach of encoding multiple qubits per photon, becomes a key enabler for pushing the limits of data rates, paving the way for more sophisticated quantum communication applications, such as, distributed quantum computations.

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Data availability. All relevant data are available within the article and its Supplement 1.

Supplemental document. See Supplement 1 for supporting content.

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