Communication over entanglement-breaking channels with unreliable entanglement assistance

Uzi Pereg [*]
ECE Department, Technion, Israel Institute of Technology, Haifa 3200003, Israel

(Received 30 May 2023; accepted 27 September 2023; published 23 October 2023)

Entanglement assistance can improve communication rates significantly. Yet its generation is susceptible to failure. The unreliable assistance model accounts for those challenges. Previous work provided an asymptotic formula that outlined the tradeoff between the unassisted and excess rates from entanglement assistance. We derive a full characterization for entanglement-breaking channels and show that combining entanglement-assisted and unassisted coding is suboptimal. From a networking perspective, this finding is nontrivial and highlights a quantum behavior arising from superposition.

DOI: 10.1103/PhysRevA.108.042616

I. INTRODUCTION

Quantum entanglement has the potential to revolutionize communication systems, as it can be used to transmit information at speeds far beyond what is possible classically [1–3]. In optical communications, generating preshared entanglement between the transmitter and the receiver can be challenging due to photon absorption during transmission. Therefore, practical systems rely on a back channel to confirm successful entanglement generation [4]. However, this introduces delays and further degrades entanglement resources. The author, along with Deppe and Boche [5], proposed an alternative approach for communication with unreliable entanglement assistance. Our principle of operation provides reliability by design, by adapting the communication rate based on the availability of entanglement assistance, while eliminating the need for feedback, repetition, or distillation.

A fundamental task in information theory is to determine the channel capacity, i.e., the ultimate transmission rate of communication with a vanishing probability of decoding error. The Holevo-Schumacher-Westmoreland (HSW) theorem provides an asymptotic description of the capacity of a quantum channel in the form of a multiletter regularized expression [6,7]. One may employ the HSW theorem to compute lower bounds on the capacity and even obtain a complete characterization in specific examples. However, in Shannon theory, multiletter capacity formulas are generally considered an incomplete solution, for reasons of computability [8], uniqueness [9], and insights on optimal coding [10]. In the entanglement-assisted communication setting, where preshared entanglement resources are available to the transmitter and the receiver, a complete single-letter characterization is well established [11] and can be viewed as the quantum parallel of Shannon's capacity theorem [12]. Therefore, entanglement-assisted communication has favorable attributes from both performance and analysis perspectives.

Let us now consider communication with *unreliable* entanglement assistance. Suppose that Alice wishes to send two messages at rates R and R'. She encodes both messages using her share of the entanglement resources, as she does not know whether Bob will have access to the entangled resources. Nevertheless, heralded entanglement generation guarantees that Bob knows whether the procedure was successful or not. Bob has two decoding procedures. If the entanglement assistance has failed to reach Bob's location, he performs a decoding operation to recover the first message alone. Hence, the communication system operates on a rate R. Whereas if Bob has entanglement assistance, he decodes both messages, hence the overall transmission rate is $R + R'$. In other words, R is a guaranteed rate and R' is the excess rate of information that entanglement assistance provides.

The previous work [5] established an asymptotic regularized formula for the capacity region, i.e., the set of all rate pairs (R, R') that can be achieved with a vanishing probability of decoding error. The achievability scheme is inspired by the classical network technique of superposition coding (SPC). We refer to the quantum method as *quantum SPC*. The classical technique consists of layered codebooks, by which the codewords are divided into so-called cloud centers and satellites, representing the first and second layers, respectively. In analogy, quantum SPC uses conditional quantum operations that map quantum cloud centers to quantum satellite states. Decoding is performed in two stages. First, Bob recovers the cloud index, corresponding to the guaranteed information. If the entanglement assistance is absent, then Bob quits after the first step. Otherwise, if Bob has entanglement assistance, then he continues to decode the satellite, i.e., the excess information. Until now, it has remained unclear whether quantum SPC is optimal.

Entanglement breaking is a fundamental property of a large class of quantum channels, mapping any entangled state to a separable state [13]. One example is the qubit depolarizing channel, which is entanglement breaking only when the depolarization parameter is greater than or equal to 2/3 [14]. From a Shannon-theoretic perspective, entanglement-breaking channels are much better understood, compared to general quantum channels [15–20]. In particular, the

[*]Also at Helen Diller Quantum Center, Technion; uzipereg@technion.ac.il

unassisted capacity is characterized by the single-letter Holevo information [15]. While an entanglement-breaking channel cannot be used to generate entanglement, it may facilitate the transmission of classical messages, and entanglement assistance can increase the channel capacity for sending classical information substantially [3]. Shor [15] established the single-letter characterization of the unassisted capacity by first showing that the Holevo information of an entanglement-breaking channel is additive. The author [21] has recently pointed out a more direct approach, proving a single-letter converse proof "from scratch."

Entanglement breaking channels and their properties have been extensively studied in the literature [22–26]. Matsumoto *et al.* [27] portrayed the relation between the additivity property and the entanglement of formation. Wilde *et al.* [18] proved the strong converse property for entanglement-breaking channels. Entanglement-breaking multiple-access channels and broadcast channels were considered in [28] and [19], respectively. More recently, Müller-Hermes and Singh [29] showed that, if the positive partial transposition (PPT) condition holds for both the channel and its complementary, then the channel is entanglement breaking, and thus antidegradable (see also [30,31]).

In this work, we establish full characterization of the capacity region with unreliable entanglement assistance for the class of entanglement-breaking channels. Our main contribution is thus a converse result that complements the previous achievability proof, and shows that quantum SPC is indeed optimal for the class of entanglement-breaking channels. The analysis relies on observations from another work by the author [21], Sec. III-D along with the geometric properties of the rate region. To complete the characterization, we single-letterize our capacity formula and show that the auxiliary systems have bounded dimensions.

We also demonstrate our results for an entanglement-breaking depolarizing channel. We show that quantum SPC can outperform time division even in this simple point-to-point setting. This is surprising because SPC is typically useful in more complex network setups, and does not yield an advantage in point-to-point communication. For example, in a classical broadcast channel with degraded messages, where a transmitter communicates with two receivers, SPC is unnecessary when the receivers' outputs are identical, as the capacity region can be attained using a simpler approach of time division. That is, concatenating two single-user codes is optimal. In our context, the system can be regarded as a quantum broadcast channel with degraded messages where one receiver has entanglement assistance and the other does not. Nevertheless, the output states of the receivers are identical (without violating the no-cloning theorem, as we consider two alternative scenarios). The expectation would be that time division, combining assisted and unassisted codes, achieves optimality. However, this expectation is proven false as quantum SPC can outperform time division, based on the combination of a superposition code with a superposition state.

Illustrative metaphor

Communication with unreliable entanglement assistance is not a mere combination of the entanglement-assisted and unassisted settings. The protocol poses a challenge as Alice must encode without knowledge of the availability of assistance. The availability of entanglement is not associated with a probabilistic model either. To illustrate the concept of reliability, consider the following metaphor.

Imagine there are N travelers embarking on a journey aboard a ship that may have a variable number of lifeboats. The total capacity of the lifeboats is L, which determines how many travelers can be accommodated in case of a shipwreck, $L \leqslant N$. The ship's speed is denoted as $V \equiv V(N, L)$, while the lifeboats' speed is v_0. If the ship does not sink, each traveler will travel at speed V. To avoid a morbid narrative, let us envision that, in the event of an unforeseen shipwreck, $(N - L)$ travelers will be safely rescued and brought back to the starting point, while the journey continues with the remaining travelers aboard the lifeboats. The speed of travel in this scenario is calculated as the average speed of the lifeboats, $R = (L/N)v_0$.

In our metaphor, R represents the guaranteed speed for the remaining travelers, while $R' = V - R$ indicates the excess speed that the ship would have provided. Increasing the number of lifeboats improves the guaranteed speed but reduces the excess speed, while decreasing the number of lifeboats has the opposite effect. When planning for the worst-case scenario, it is crucial to consider both speeds, R and R', rather than just the average speed.

One may consider the option of dividing the travelers among a heavy ship and a light ship. Figuratively, our findings show that if the journey is subject to a quantum evolution, then we may outperform the division plan by allowing travelers to be in a quantum superposition state between the two ships.

II. CODING WITH UNRELIABLE ASSISTANCE

A. Notation, information measures, and quantum channels

We use standard notation for quantum channels and information measures, as in [9], Chap. 11. The letters $X, Y, Z, \dots,$ represent discrete random variables, on finite sets $\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \dots$, respectively. The distribution of X is specified by a probability mass function (pmf) $p_X(x)$ on $\mathcal{X}$. We use $x^n = (x_i)_{i\in[n]}$ to denote a sequence of letters from $\mathcal{X}$.

The state of a quantum system A is given by a density operator on the Hilbert space $\mathcal{H}_A$. A measurement is specified by a collection of operators $\{D_j\}$ that forms a positive operator-valued measure (POVM), i.e., $D_j \geqslant 0$ and $\sum_j D_j = \mathbb{1}$, where $\mathbb{1}$ is the identity operator. Given a bipartite state ρ_{AB}, define the quantum mutual information by $I(A;B)_\rho = H(\rho_A) + H(\rho_B) - H(\rho_{AB})$, where $H(\rho) \equiv -\text{Tr}[\rho \log_2(\rho)]$ is the von Neumann entropy. The conditional quantum entropy and mutual information are defined by $H(A|B)_\rho = H(\rho_{AB}) - H(\rho_B)$ and $I(A;B|C)_\rho = H(A|C)_\rho + H(B|C)_\rho - H(A,B|C)_\rho$, respectively.

A quantum channel $\mathcal{N}_{A\to B}$ is a completely-positive trace-preserving (cptp) map. If the systems $A^n = (A_1, \dots, A_n)$ are sent through n channel uses, then the input state ρ_{A^n} undergoes the tensor product mapping $\mathcal{N}^{\otimes n}_{A\to B}$. The channel is called entanglement breaking if, for every input state $\rho_{AA'}$, where A' is an arbitrary reference system, the channel output is separable, i.e., $(\mathcal{N}_{A\to B} \otimes \mathbb{1})(\rho_{AA'}) = \sum_{x\in\mathcal{X}} p_X(x)\psi^x_B \otimes \psi^x_{A'}$, for some pmf p_X and pure states ψ_B^x and $\psi_{A'}^x$. The Kraus representation of an entanglement-breaking channel consists of unit-rank Kraus operators. Furthermore, every entanglement-breaking channel can be represented as a serial concatenation of a measurement channel followed by a classical-quantum channel [9], Corollary 4.6.1.

B. Coding and channel capacity

We define a code for communication with unreliable entanglement resources. Alice and Bob's entangled systems are denoted T_A and T_B, respectively.

Definition 1. A $(2^{nR}, 2^{nR'}, n)$ code with unreliable entanglement assistance consists of the following: Two message sets $[2^{nR}]$ and $[2^{nR'}]$, where 2^{nR}, $2^{nR'}$ are integers, an entangled state Ψ_{T_A,T_B}, a collection of encoding maps $\mathcal{F}^{m,m'}_{T_A\to A^n}$ for $m \in [2^{nR}]$ and $m' \in [2^{nR'}]$, and two decoding POVMs, $\mathcal{D}_{B^nT_B} = \{D_{m,m'}\}$ and $\mathcal{D}^*_{B^n} = \{D^*_m\}$.

Alice chooses two messages, $m \in [2^{nR}]$ and $m' \in [2^{nR'}]$. She applies the encoding map to her share of the entangled state, and then transmits A^n over n channel uses of $\mathcal{N}_{A\to B}$. Bob receives B^n. If the entanglement assistance is present, i.e., Bob has access to the resource T_B, then he should recover both messages. He performs a joint measurement $\mathcal{D}_{B^nT_B}$ to obtain an estimate $(\hat{m}, \hat{m}')$.

Otherwise, if entanglement assistance is absent, Bob does not have T_B. Hence, he performs the measurement $\mathcal{D}^*_{B^n}$ to obtain an estimate $\hat{\hat{m}}$ of the first message alone. The error probability is

$$P^{(n)}_{e|m,m'} = 1 - \mathrm{Tr}\big[\mathcal{D} \circ \mathcal{N}^{\otimes n}_{A\to B} \circ \mathcal{F}^{m,m'}(\Psi_{T_A,T_B})\big] \quad (1)$$

in the presence of entanglement assistance, and

$$P^{*(n)}_{e|m,m'} = 1 - \mathrm{Tr}\big[\mathcal{D}^* \circ \mathcal{N}^{\otimes n}_{A\to B} \circ \mathcal{F}^{m,m'}(\Psi_{T_A})\big] \quad (2)$$

without assistance. The encoded input remains the same in both scenarios since Alice does not know whether entanglement is available or not. Therefore, the error depends on (m, m') in both cases. A rate pair (R, R') is achievable if there exists a sequence of $(2^{nR}, 2^{nR'}, n)$ codes with unreliable entanglement assistance, such that $\max(P^{(n)}_{e|m,m'}, P^{*(n)}_{e|m,m'}) \to 0$ as $n \to \infty$. The capacity region $\mathcal{C}_{\mathrm{EA}^*}(\mathcal{N})$ with unreliable entanglement assistance is defined as the set of achievable rate pairs.

III. RESULTS

Let $\mathcal{N}_{A\to B}$ be an entanglement-breaking channel (see Sec. II A). Define the region

$$\mathcal{R}_{\mathrm{EA}^*}(\mathcal{N}) = \bigcup \left\{ \begin{array}{rl} (R, R') : R \leqslant & I(X;B)_\omega \\ R' \leqslant & I(G_2;B|X)_\omega \end{array} \right\}, \quad (3)$$

where the union is over all auxiliary variables $X \sim p_X$, all quantum states $\varphi_{G_1G_2}$, and all encoding channels $\mathcal{F}^{(x)}_{G_1\to A}$,

$$\omega_{XAG_2} = \sum_{x\in\mathcal{X}} p_X(x)|x\rangle\langle x| \otimes \big(\mathcal{F}^{(x)}_{G_1\to A} \otimes \mathrm{id}\big)(\varphi_{G_1G_2}),$$
$$\omega_{XBG_2} = (\mathrm{id} \otimes \mathcal{N}_{A\to B} \otimes \mathrm{id})(\omega_{XAG_2}), \quad (4)$$

where id is the identity map. Intuitively, X represents the guaranteed information, and G_1, G_2 are Alice and Bob's resources. Since the entangled resources G_1 and G_2 are preshared, the state is uncorrelated with the messages. Alice encodes the excess information using the encoding channel $\mathcal{F}^{(x)}$.

A. Capacity theorem

Our main results are stated below, characterizing the capacity region for communication over entanglement-breaking channels with unreliable entanglement assistance. Previous work [5] established a regularized characterization for the capacity region, i.e., an asymptotic *multiletter formula* of the form $\bigcup_{K=1}^{\infty} \frac{1}{K}\mathcal{R}(\mathcal{N}^{\otimes K})$. Here, we provide a complete characterization in the form of a single-letter formula.

Theorem 1. The capacity region of an entanglement-breaking quantum channel $\mathcal{N}_{A\to B}$ with unreliable entanglement assistance is given by

$$\mathcal{C}_{\mathrm{EA}^*}(\mathcal{N}) = \mathcal{R}_{\mathrm{EA}^*}(\mathcal{N}), \quad (5)$$

where $\mathcal{R}_{\mathrm{EA}^*}(\mathcal{N})$ is as defined in (3).

The proof is given in Sec. IV B.

Remark 1. Single-letterization is highly valued in Shannon theory for reasons of computability [8], uniqueness [9], and insights on optimal coding [10]. See further discussion in Sec. VI B. However, the result in Theorem 1 in itself is not enough to claim that this is truly a single-letter characterization, as the computation of a rate region requires specified dimensions. Thereby, we show in Sec. III C that the auxiliary systems, X, G_1, and G_2, all have bounded dimensions. Together, the results in Theorem 1 and Sec. III C complete the characterization.

B. Equivalent characterization

Before we prove the capacity theorem, we establish useful properties of the region $\mathcal{R}_{\mathrm{EA}^*}(\mathcal{N})$, as defined in (3). We show an equivalence to the region below

$$\mathcal{O}_{\mathrm{EA}^*}(\mathcal{N}) = \bigcup \left\{ \begin{array}{rl} (R, R') : R \leqslant & I(X;B)_\omega \\ R + R' \leqslant & I(XG_2;B)_\omega \end{array} \right\}, \quad (6)$$

where the union is as in (3). This will be useful in the proof for our main theorem in Sec. IV B, where we will show that every achievable rate pair must lie within $\mathcal{O}_{\mathrm{EA}^*}(\mathcal{N})$.

We give the intuition below. To show the equivalence between the regions $\mathcal{R}_{\mathrm{EA}^*}(\mathcal{N})$ and $\mathcal{O}_{\mathrm{EA}^*}(\mathcal{N})$, we use the geometric properties of our regions, as illustrated in Fig. 1. The region $\mathcal{R}_{\mathrm{EA}^*}(\mathcal{N})$ is defined in (3) as a union of rectangles. In particular, the light-shaded rectangle in Fig. 1 corresponds to the bounds $0 \leqslant R \leqslant I(X;B)_\omega$ and $0 \leqslant R' \leqslant I(G_2;B|X)_\omega$, for a fixed auxiliary variable, state, and encoding channels. The corner point of the region is denoted by $P_0 = [I(X;B)_\omega, I(G_2;B|X)_\omega]$. Similarly, the region $\mathcal{O}_{\mathrm{EA}^*}(\mathcal{N})$ is a union of trapezoids, the corners of which are P_0 and $P_1 = [0, I(XG_2;B)_\omega]$. Hence, the dark shaded area in Fig. 1 is the gap between the rectangle and the trapezoid, which the regions $\mathcal{R}_{\mathrm{EA}^*}(\mathcal{N})$ and $\mathcal{O}_{\mathrm{EA}^*}(\mathcal{N})$ are comprised of. Now, observe

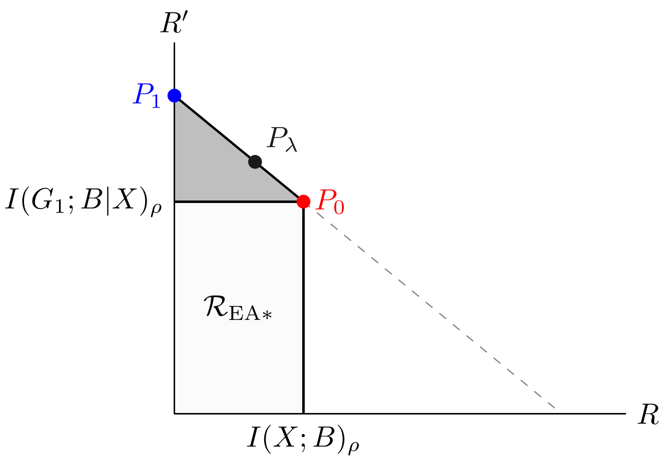

FIG. 1. Achievable rate regions.

that the point P_1 belongs to *another* rectangle in $\mathcal{R}_{\rm EA*}(\mathcal{N})$, taking $\bar{G}_2$ and $\bar{X}$ to be (X, G_2) and null, respectively. Therefore, the convexity of the region $\mathcal{R}_{\rm EA*}(\mathcal{N})$ implies that any convex combination $P_\lambda = (1-\lambda)P_0 + \lambda P_1$ must also lie within $\mathcal{R}_{\rm EA*}(\mathcal{N})$, thereby $\mathcal{R}_{\rm EA*}(\mathcal{N}) = \mathcal{O}_{\rm EA*}(\mathcal{N})$. The details are given below.

We begin with the convexity of our original region.

Lemma 1. The rate region $\mathcal{R}_{\rm EA*}(\mathcal{N})$ is a convex set.

As explained above, the convexity of $\mathcal{R}_{\rm EA*}(\mathcal{N})$ implies that the point P_λ in Fig. 1 is included within the union of rectangles, i.e., $P_\lambda \in \mathcal{R}_{\rm EA*}(\mathcal{N})$. We obtain the following consequence.

Corollary 1. For every $\lambda \in [0, 1]$,

$$\mathcal{R}_{\rm EA*}(\mathcal{N}) \supseteq \left\{ \begin{matrix} (R, R') \,:\, R \leqslant & (1-\lambda)I(X;B)_\omega \\ R' \leqslant & I(G_2;B|X)_\omega + \lambda I(X;B)_\omega \end{matrix} \right\}. \quad (7)$$

The proof for the convexity properties in Lemma 1 and Corollary 1 is given in Appendix. Next, we use those properties to establish equivalence.

Lemma 2 (Equivalence). $\mathcal{R}_{\rm EA*}(\mathcal{N}) = \mathcal{O}_{\rm EA*}(\mathcal{N})$.

Proof. The inclusion $\mathcal{R}_{\rm EA*}(\mathcal{N}) \subseteq \mathcal{O}_{\rm EA*}(\mathcal{N})$ is immediate by the chain rule. It remains to show that every rate pair in the region $\mathcal{O}_{\rm EA*}(\mathcal{N})$, belongs to $\mathcal{R}_{\rm EA*}(\mathcal{N})$ as well.

Let $(R, R') \in \mathcal{O}_{\rm EA*}(\mathcal{N})$, hence

$$R \leqslant I(X;B)_\omega, \;\; R + R' \leqslant I(XG_2;B)_\omega. \quad (8)$$

By the first inequality, there exists $0 \leqslant \lambda \leqslant 1$ such that

$$R = (1-\lambda)I(X;B)_\omega. \quad (9)$$

By (8) and (9),

$$\begin{aligned} R' &\leqslant I(XG_2;B)_\omega - R \\ &= I(XG_2;B)_\omega - I(X;B)_\omega + \lambda I(X;B)_\omega \\ &= I(G_2;B|X)_\omega + \lambda I(X;B)_\omega. \end{aligned} \quad (10)$$

Hence, by Corollary 1, $(R, R') \in \mathcal{R}_{\rm EA*}(\mathcal{N})$. ■

C. Single letterization

As mentioned above, single letterization is highly valued in Shannon theory (see Remark 1 and discussion in Sec. VI B). We establish that our characterization is a single-letter formula. Specifically, the auxiliary systems, X, G_1, and G_2, all have bounded dimensions. Denote the channel input dimension by $d_A \equiv \dim(\mathcal{H}_A)$.

Lemma 3. The union in (3) is exhausted by pure states $|\phi_{G_1G_2}\rangle$, cardinality $|\mathcal{X}| \leqslant d_A^2 + 1$, and dimensions $\dim(\mathcal{H}_{G_1}) = \dim(\mathcal{H}_{G_2}) \leqslant d_A(d_A^2 + 1)$.

The first part has already been stated in [5]. The quantum dimension bound is new, see the proof in Sec. IV A below.

IV. ANALYSIS

A. Single letterization

The first part of Lemma 3 has already been established in our previous work [5], Lemma 2, using convex analysis. Bounding the quantum dimensions is more challenging.

Consider a pure state $|\psi_{G_1G_2}\rangle$. Since the Schmidt rank is bounded by each dimension, we may assume without loss of generality (w.l.o.g.) that G_1 and G_2 are qudits of the same dimension d_0, for some $d_0 > 0$. We would like to show that the union can be restricted such that encoded state $\omega^x_{G_2A} \equiv (\mathrm{id} \otimes \mathcal{F}^{(x)}_{G_1A})(|\psi_{G_2G_1}\rangle\langle\psi_{G_2G_1}|)$ remains pure.

Since every quantum channel has a Stinespring dilation, there exists a unitary $V^{(x)}$ such that $\mathcal{F}^{(x)}_{G_1\to A}(\rho) = \mathrm{Tr}_{DE}[V^{(x)}(|0\rangle\langle 0|_D \otimes \rho)V^{(x)\dagger}]$, where $V^{(x)}$ maps from $\mathcal{H}_D \otimes \mathcal{H}_{G_1}$ to $\mathcal{H}_E \otimes \mathcal{H}_A$, while D, E are reference systems with appropriate dimensions. Since G_1 is an arbitrary ancilla, we may include the reference D within this ancilla, and simplify as $\mathcal{F}^{(x)}_{G_1\to A}(\rho) = \mathrm{Tr}_E[U^{(x)}\rho U^{(x)\dagger}]$, where $U^{(x)}$ is a unitary from $\mathcal{H}_{G_1}$ to $\mathcal{H}_E \otimes \mathcal{H}_A$.

We would like the ancilla G_2 to absorb the reference E as well. Seemingly, this would contradict (3) as E could be correlated with x. To resolve this difficulty, we show that the encoding operation can be reflected to G_2. Fix $x \in \mathcal{X}$ and consider the purification $|\omega^{(x)}_{G_2EA}\rangle \equiv (\mathbb{1} \otimes U^{(x)})|\psi_{G_2G_1}\rangle$.

Let $W_{i,j}$ denote the Weyl operators on $\mathcal{H}_{G_1} \cong \mathcal{H}_{G_2}$, for $i, j \in \{0, \ldots, d_0 - 1\}$ [9], Sec. 3.7.2. By plugging a decomposition of $|\psi_{G_2G_1}\rangle$ in the generalized Bell basis [9], Ex. 3.7.11, and applying the mirror lemma, by which $(\mathbb{1} \otimes U)|\Phi\rangle = (U^T \otimes \mathbb{1})|\Phi\rangle$ for every qudit operator U [9], Ex. 3.7.12, we obtain $|\omega^{(x)}_{G_2EA}\rangle = \sum_{i,j=0}^{d_0-1} \alpha_{i,j}(W_{i,j}F^{(x)}_{G_1\to G_2E} \otimes \mathbb{1}_A)|\Phi\rangle_{G_1A}$, with $F^{(x)}_{G_1\to G_2E} = (U^{(x)})^T$. We see that (3) can thus be represented as a union over all unitaries $F^{(x)}_{G_1\to G_2E} \otimes \mathbb{1}_A$.

In this formulation, both E and G_2 are encoded by an operation depending on x. Thus, we can extend the union to $\bar{G}_2 = (G_2, E)$. The bound on the guaranteed rate R remains. As for the excess rate, $I(\bar{G}_2;B|X)_\omega \geqslant I(G_2;B|X)_\omega$. Hence, it suffices to consider pure states $|\omega^{(x)}_{G_2A}\rangle$, the Schmidt rank of which is bounded by d_A. Thus, the region is exhausted with $d_0 \leqslant |\mathcal{X}|d_A$. ■

B. Capacity proof

The direct part was proved in our earlier work [5]. We now focus our attention on the converse. Suppose that Alice and Bob share an unreliable resource $\Psi_{T_AT_B}$. Alice first prepares

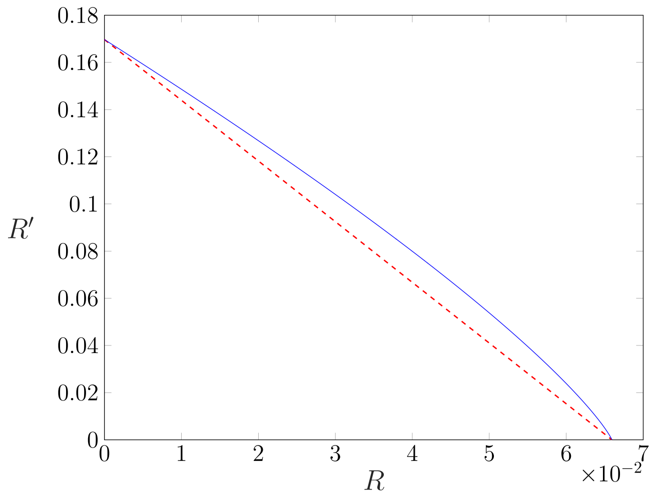

FIG. 2. Achievable rate regions.

the classical correlation

$$\pi_{KMK'M'} \equiv \left(\frac{1}{2^{nR}}\sum_{m=1}^{2^{nR}}|m\rangle\langle m|\otimes|m\rangle\langle m|\right) \otimes\left(\frac{1}{2^{nR'}}\sum_{m'=1}^{2^{nR'}}|m'\rangle\langle m'|\otimes|m'\rangle\langle m'|\right) \tag{11}$$

locally. She encodes by $\mathcal{F}_{MM'T_A\to A^n}$, and transmits A^n. Bob receives B^n in the state $\omega_{KK'T_BB^n}\equiv(\mathrm{id}\otimes\mathcal{N}^{\otimes n}\mathcal{F})(\pi\otimes\Psi)$. He decodes with either $\mathcal{D}_{B^nT_B\to\hat{M}\hat{M}'}$ or $\mathcal{D}^*_{B^n\to\tilde{M}}$, depending on the availability of entanglement assistance.

Consider a sequence of codes $(\mathcal{F}_n, \Psi_n, \mathcal{D}_n, \mathcal{D}^*_n)$ with vanishing errors. By continuity and data processing arguments [5, Appendix C]

$$nR \leqslant I(K;B^n)_\omega + n\varepsilon^*_n, \tag{12}$$

$$n(R+R') \leqslant I(KK'T_B;B^n)_\omega + n\varepsilon_n, \tag{13}$$

where $\varepsilon_n, \varepsilon^*_n \to 0$ as $n\to\infty$.

Since the channel is entanglement breaking, it can be represented by a measurement channel $\mathcal{M}_{A\to Y}$, followed by a preparation channel $\mathcal{P}_{Y\to B}$, where Y is classical [21], Sec. III-D. Define the sequence of classical variables, $X_i\equiv(K,Y^{i-1})$, for $i\in[n]$. By the chain rule and the data processing inequality, (12) and (13) imply

$$\begin{aligned} n(R-\varepsilon^*_n) &\leqslant \sum_{i=1}^n I(KB^{i-1};B_i)_\omega \\ &\leqslant \sum_{i=1}^n I(KY^{i-1};B_i)_\omega \\ &= \sum_{i=1}^n I(X_i;B_i)_\omega, \end{aligned} \tag{14}$$

and similarly,

$$n(R+R'-\varepsilon_n) \leqslant \sum_{i=1}^n I(K'T_BX_i;B_i)_\omega. \tag{15}$$

Letting J be uniformly distributed index in $[n]$, we have $R-\varepsilon^*_n \leqslant I(X_J;B_J|J)_\omega \leqslant I(JX_J;B_J)_\omega$ and $R+R'-\varepsilon_n \leqslant I(K'T_BJX_J;B_J)_\omega$ with respect to $\omega_{JK'T_BX_JB_J}\equiv\frac{1}{n}\sum_{i=1}^n |i\rangle\langle i|_J\otimes\omega_{K'T_BX_iB_i}$.

Taking $G_2\equiv(K',T_B)$, $X\equiv(J,X_J)$, $A\equiv A_J$, hence $B\equiv B_J$, we deduce that $(R,R')\in\mathcal{O}_{EA*}(\mathcal{N})$. This, in turn, implies $(R,R')\in\mathcal{R}_{EA*}(\mathcal{N})$, by Lemma 2. ■

V. EXAMPLE

Consider the qubit depolarizing channel, $\mathcal{N}(\rho)=(1-\varepsilon)\rho+\varepsilon\frac{\mathbb{1}}{2}$, with $\varepsilon\in[0,1]$. The unassisted capacity, $C(\mathcal{N})$, is achieved with a symmetric distribution over $\{|0\rangle,|1\rangle\}$ (see [32]). On the other hand, the capacity with reliable entanglement assistance $C_{\text{EA}}(\mathcal{N})$ is achieved with an EPR state [33]. A classical mixture of those strategies yields the time division region, $\mathcal{C}_{\text{EA}*}(\mathcal{N}) \supseteq \bigcup_{0\leqslant\lambda\leqslant1}\left\{{}^{(R,R')\,:\,R\leqslant\,(1-\lambda)C(\mathcal{N})}_{\qquad\quad R'\leqslant\,\lambda C_{\text{EA}}(\mathcal{N})}\right\}$. We claim that this is suboptimal.

Figure 2 depicts the capacity region for a parameter such that the channel is entanglement breaking, $\varepsilon=0.7$ (as opposed to [5], Example 1). The time-division bound is below the red line, whereas the blue curve indicates the capacity region that is achieved using a superposition state. Based on Theorem 1, we establish that the capacity region of an entanglement-breaking qubit depolarizing channel with unreliable entanglement assistance is given by

$$\mathcal{C}_{\text{EA}*}(\mathcal{N}) = \bigcup_{0\leqslant\alpha\leqslant\frac12}\left\{\begin{aligned}(R,R')\,:\,R &\leqslant 1-h_2\left(\alpha*\tfrac{\varepsilon}{2}\right)\\ R' &\leqslant h_2(\alpha)+h_2\left(\alpha*\tfrac{\varepsilon}{2}\right)-H\Big[\tfrac{\alpha\varepsilon}{2},\tfrac{(1-\alpha)\varepsilon}{2},\\ &\tfrac12-\tfrac{\varepsilon}{4}-\sqrt{\tfrac{\varepsilon^2}{16}-(1-\alpha)\alpha\varepsilon\left(1-\tfrac{3\varepsilon}{4}\right)+\tfrac{1-\varepsilon}{4}},\\ &\tfrac12-\tfrac{\varepsilon}{4}+\sqrt{\tfrac{\varepsilon^2}{16}-(1-\alpha)\alpha\varepsilon\left(1-\tfrac{3\varepsilon}{4}\right)+\tfrac{1-\varepsilon}{4}}\Big]\end{aligned}\right\}, \tag{16}$$

where $H(\mathbf{p})\equiv-\sum_i p_i\log_2(p_i)$ is the Shannon entropy for a classical probability vector $\mathbf{p}$, the binary entropy function is denoted by $h_2(x)\equiv H(x,1-x)$ for $x\in[0,1]$, and $\alpha*\beta=(1-\alpha)\beta+\alpha(1-\beta)$ is the binary convolution operation.

Proof. By Theorem 1, it suffices to evaluate the region $\mathcal{R}_{\text{EA}*}(\mathcal{N})$, as defined in (3).

We begin with the converse part and show that the set on the right-hand side of (16) is an outer bound on $\mathcal{R}_{\text{EA}*}(\mathcal{N})$.

Consider a rate pair $(R, R') \in \mathcal{R}_{\text{EA*}}(\mathcal{N})$. Hence, $R \leqslant I(X;B)_\omega$ and $R' \leqslant I(G_2;B|X)_\omega$, or, equivalently,

$$R \leqslant H(B)_\omega - H(B|X)_\omega, \tag{17a}$$

$$R' \leqslant H(G_2|X)_\omega + H(B|X)_\omega - H(G_2B|X)_\omega, \tag{17b}$$

for some pure input state $|\phi_{G_1G_2}\rangle$, variable $X \sim p_X$, and encoder $\mathcal{F}^{(x)}_{G_1\to A}$ (see Lemma 3).

Based on the analysis in Sec. IV A, it suffices to consider an encoder that produces a pure state $|\omega^{(x)}_{G_2A}\rangle$, for $x \in \mathcal{X}$. Consider a Schmidt decomposition

$$\left|\omega^{(x)}_{G_2A}\right\rangle = \sqrt{1-\alpha_x}|\theta_{0x}\rangle \otimes |\psi_{0x}\rangle + \sqrt{\alpha_x}|\theta_{1x}\rangle \otimes |\psi_{1x}\rangle,$$

with $\alpha_x \in [0, 1]$. Since the encoding channel is applied to G_1 alone, the reduced state of G_2 remains unchanged. Thereby, the eigenvalues $(1-\alpha_x, \alpha_x)$ must be independent of x. That is, $\alpha_x \equiv \alpha$ for $x \in \mathcal{X}$, hence

$$H(G_2|X)_\omega = h_2(\alpha). \tag{18}$$

Furthermore, the depolarizing channel is unitarily covariant, i.e., $\mathcal{N}(U\rho U^\dagger) = U\mathcal{N}(\rho)U^\dagger$ for every unitary U on $\mathcal{H}_A$. Thus,

$$H(B|X)_\omega = H[\mathcal{N}(\widetilde{\phi}_A)] = h_2\left(\alpha * \frac{\varepsilon}{2}\right), \tag{19}$$

where $|\widetilde{\phi}_{G_2A}\rangle = (1-\alpha)|00\rangle + \alpha|11\rangle$, and similarly,

$$\begin{aligned} H(G_2B|X)_\omega &= H((\mathrm{id}\otimes\mathcal{N})(\widetilde{\phi}_{G_2A})) \\ &= H\Bigg(\frac{\alpha\varepsilon}{2}, \frac{(1-\alpha)\varepsilon}{2}, \frac{1}{2} - \frac{\varepsilon}{4} \\ &\quad \pm \sqrt{\frac{\varepsilon^2}{16} - (1-\alpha)\alpha\varepsilon\left(1-\frac{3\varepsilon}{4}\right) + \frac{1-\varepsilon}{4}}\Bigg) \end{aligned} \tag{20}$$

(see [34]). As the output entropy is bounded by $H(B)_\omega \leqslant 1$, the converse follows from (17) to (20).

Achievability follows as in [5], Example 1. Instead of a classical mixture, we now use quantum superposition. Set $|\phi_{G_1G_2}\rangle \equiv \sqrt{1-\alpha}|00\rangle + \sqrt{\alpha}|11\rangle$, $p_X = (\frac{1}{2}, \frac{1}{2})$, $\mathcal{F}^{(x)}(\rho) \equiv \mathsf{X}^x\rho\mathsf{X}^x$, where X is the bit-flip Pauli operator. Thus, $\alpha = 0$ and $\alpha = \frac{1}{2}$ achieve the unassisted capacity and entanglement-assisted capacity, respectively. The resulting region is the set on the right-hand side of (16). ■

VI. SUMMARY AND DISCUSSION

We address communication over an entanglement-breaking quantum channel, given *unreliable* entanglement assistance. Previous work established a multiletter asymptotic formula and presented the quantum "superposition coding" (SPC) achievable region [5]. Here, we show that the region is optimal for entanglement-breaking channels, and we single-letterize the formula, providing a complete characterization of the capacity region. Furthermore, we derive a closed-form expression for the qubit depolarizing channel, with a parameter $\varepsilon \geqslant \frac{2}{3}$. It is further demonstrated that the capacity region is strictly larger than the time-division rate region. From a networking perspective, this finding is nontrivial and highlights a quantum behavior arising from superposition.

We conclude with a discussion on the application in a dynamic communication network, the importance of single-letterization, the role of entanglement breaking channels, and the challenges posed by unreliable entanglement resources: the underlying motivations, the concept of "hard decision decoding," the links to classical models, surprising behavior, and the expected impact.

A. Dynamic communication and entanglement resources

In a dynamic communication network, information is not necessarily transmitted between two particular nodes at every point in time. In principle, in the "quiet" period of time, entanglement can be generated between those nodes. While entanglement can be harnessed to generate shared randomness, its potential utility extends far beyond that [35,36]. This motivates using entanglement to enhance various communication networks and applications, such as the Internet of Things (IoT) [37–40].

Superdense coding [41] is a fundamental communication protocol, where a pair of classical bits is transmitted using just one instance of a noiseless qubit channel and a maximally entangled pair. This means that entanglement assistance effectively doubles the rate at which classical messages can be sent over a noiseless qubit channel.

B. Single letterization

In Shannon theory, the efficiency of communication across noisy channels is described by the concept of channel capacity. The capacity is defined as the maximum transmission rate that permits an error probability that tends to zero in the limit of an infinite blocklength. Remarkably, Shannon [12] proved that the capacity of a classical channel $W_{Y|X}$ admits a single-letter formula, i.e., a nonasysmtotic expression. The significance of such single letterization is attributed to the following:

(1) *Computability.* Shannon's capacity formula is generally considered to be "easy to compute" in the sense that given the channel statistics, there are efficient algorithms, such as the Blahut-Arimoto algorithm [42,43] that can solve this convex optimization problem numerically, up to a given precision and provided that the input and output dimensions are not too large. On the other hand, a multiletter formula, of the form

$$\lim_{n\to\infty} \frac{1}{n} f(W^{\otimes n}), \tag{21}$$

is difficult to compute since the dimensions of $W^{\otimes n}$ grow exponentially with n.

(2) *Uniqueness.* A multiletter formula does not uniquely characterize the capacity of a channel for a given task [9]. For instance, the capacity of a classical channel can be expressed as [9], Sec. 13.1.3

$$\lim_{n\to\infty} \frac{1}{n} f_c(W^{\otimes n}) = f_1(W), \tag{22}$$

where

$$f_c(W) = H(X) - cH(X|Y) \tag{23}$$

for every constant $c \geqslant 1$. The multiletter formulas $\lim \frac{1}{n} f_1(W^{\otimes n})$ and $\lim \frac{1}{n} f_5(W^{\otimes n})$ have a different form, and yet, both describe the channel capacity. Hence, such a multiletter description is not unique.

(3) *Optimal coding.* Single-letter formulas provide valuable insights into optimal coding strategies in various settings. For instance, the characterization for the multiple access channel captures coding techniques such as time sharing and successive-cancellation decoding [44–46]. For parallel Gaussian channels, the capacity formula leads to the water filling power allocation [47–49], among other applications.

Unfortunately, a single-letter characterization for the capacity of a quantum channel is an open problem [21,50]. Nevertheless, it is important to note that multiletter characterizations remain significant [51], Remark 7. In many examples, the capacity can be evaluated exactly based on the multiletter result [9,50]. Furthermore, there are interesting phenomena that can be observed even when a single-letter expression for the capacity is not available [52,53].

In the entanglement-assisted communication setting, where preshared entanglement resources are available to the transmitter and the receiver, a complete single-letter characterization is well established, and can be viewed as the quantum parallel of Shannon's capacity theorem [11] [51, Remark 5]. The characterization with entanglement assistance provides a computable upper bound for unassisted communication as well. Therefore, entanglement-assisted communication has favorable attributes from both performance and analysis perspectives.

C. Entanglement breaking channels

Entanglement breaking is a fundamental property of a large class of quantum channels (see Sec. II A), including measurement channels and classical-quantum (c-q) channels [13]. The qubit depolarizing channel is entanglement breaking if and only if the depolarization parameter is greater than or equal to 2/3 [14].

While an entanglement-breaking channel cannot be used to generate entanglement, it may facilitate the transmission of classical messages, and entanglement assistance can increase the channel capacity for sending classical information substantially [3]. Furthermore, the capacity without assistance is solved as well. Shor [15] established the single-letter characterization of the unassisted capacity by first showing that the Holevo information of an entanglement-breaking channel is additive.

Here, we used a more direct approach, which was recently pointed out by the author [21] (see also [19]), proving a single-letter converse proof "from scratch." The proof is based on the representation of an entanglement breaking channel as a serial concatenation of a measurement channel and a c-q channel, along with the data-processing inequality (see capacity proof in Sec. IV B).

D. Unreliable entanglement resources

Communication with unreliable entanglement assistance was proposed by the author, along with Deppe and Boche [5]. The model accounts for the practical challenges and low efficiency of entanglement generation in current implementations and experiments [54]. The framework is inspired by classical approaches for unreliable cooperation resources in the classical literature [55–61]. The focus in our quantum setting, however, is on a point-to-point quantum channel and the reliability of correlation resources.

Our principle of operation provides reliability by design, by adapting the communication rate based on the availability of entanglement assistance, while eliminating the need for feedback, repetition, or distillation. As illustrated in the Introduction, and as opposed to other models in the literature [62], the availability of entanglement is not associated with a probabilistic model either. Here, the receiver is aware of the availability or absence of entanglement resources through heralded entanglement generation. The receiver performs "hard decision decoding" [63], deciding whether the entanglement resources are usable or not at all.

Drawing a parallel with the classical cooperation model [56], the unreliable assistance model is based on the engineering aspects and the architecture of modern communication networks. We anticipate that future quantum communication networks will adhere to similar reliability principles. In particular, we envision that in a large quantum communication network, the availability of entanglement resources will not be guaranteed in advance. Specifically, the accessibility of entanglement resources will depend on factors such as weather conditions, the operational status of quantum repeaters, or the willingness of peers to provide assistance. In such a network, the transmitter and the receiver are aware of the *possibility* that entanglement assistance will be available, yet its confirmation remains uncertain until reception.

The model exhibits unexpected behavior, highlighting that communication with unreliable entanglement assistance is not a mere combination of the entanglement-assisted and unassisted settings. We showed that, for an entanglement-breaking depolarizing channel, quantum SPC outperforms time division even in this simple point-to-point setting. This is surprising because SPC is typically useful in more complex network setups, and does not yield an advantage in point-to-point communication. For example, in a classical broadcast channel with degraded messages, where a transmitter X communicates with two receivers, Y_1 and Y_2, SPC is unnecessary when the receivers' outputs are identical, i.e., $Y_1 = Y_2$, as the capacity region can be attained using a simpler approach of time division. That is, concatenating two single-user codes is optimal. In our context, the system can be regarded as a quantum broadcast channel with degraded messages where one receiver has entanglement assistance and the other does not. Nevertheless, the output states of the receivers are identical (without violating the no-cloning theorem, as we consider two alternative scenarios). The expectation would be that time division, combining assisted and unassisted codes, achieves optimality. However, this expectation is proven false as quantum SPC can outperform time division, based on the combination of a superposition code with a superposition state.

We expect that the present work will have a significant impact due to its relevance to practical systems, the interesting and unexpected properties, and the potential applicability of our reliability principles across a wide variety of tasks and protocols that rely on preestablished entanglement. These range between research areas such as communication, distributed computing, complexity theory, and cryptography, among others.

ACKNOWLEDGMENTS

The author thanks Christoph Becher (Universität des Saarlandes) for his observations on the practical implementation of heralded entanglement, as well as Quntao Zhuang (University of Southern California) and Saikat Guha (University of Arizona) for useful discussions on entanglement-breaking channels. This work was supported by the Israel VATAT Junior Faculty Program for Quantum Science and Technology through Grant No. 86636903, Israel Science Foundation (ISF), Grants No. 939/23 and No. 2691/23, Chaya Career Advancement Chair, Grant No. 8776026, German-Israeli Project Cooperation (DIP), Grant No. 2032991, and Nevet Program of the Helen Diller Quantum Center at the Technion.

APPENDIX: CONVEXITY PROPERTIES

In Sec. III B, we presented convexity properties of the region $\mathcal{R}_{\text{EA}*}(\mathcal{N})$, as defined in (3). Since the derivation is technical, we delegated the proof to the Appendix.

1. Proof of Lemma 1

Let $\lambda \in [0,1]$. Consider two rate pairs, (R_u, R'_u), $u \in \{1,2\}$, that belong to the rate region $\mathcal{R}_{\text{EA}*}(\mathcal{N})$. Then, we have $R_u \leqslant I(X;B|U=u)_\omega$ and $R'_u \leqslant I(G_2^{(u)};B|X,U=u)_\omega$ for some conditional distribution $p_{X|U}$, entangled state $\varphi_{G_1^{(u)}G_2^{(u)}}$, and encoding channel $\mathcal{F}^{(x,u)}_{G_1^{(u)}\to A}$.

Consider the joint state

$$\varphi_{\bar{A}_0\bar{A}_1} = \varphi_{G_1^{(1)}G_2^{(1)}} \otimes \varphi_{G_1^{(2)}G_2^{(2)}}. \tag{A1}$$

Given $u \in \{1,2\}$, define an encoding channel $\bar{\mathcal{F}}^{(x,u)}_{\bar{A}_0\to A}$ that maps from $\bar{A}_0$ to A, such that

$$\bar{\mathcal{F}}^{(x,1)}_{\bar{A}_0\to A} \equiv \mathcal{F}^{(x,1)}_{G_1^{(1)}\to A} \circ \text{Tr}_{G_1^{(2)}}, \tag{A2}$$

$$\bar{\mathcal{F}}^{(x,1)}_{\bar{A}_0\to A} \equiv \mathcal{F}^{(x,1)}_{G_1^{(1)}\to A} \circ \text{Tr}_{G_1^{(2)}}. \tag{A3}$$

The system A is then sent through the channel $\mathcal{N}_{A\to B}$. We note that if $U=1$, then the output is uncorrelated with $G_2^{(2)}$. Similarly, for $U=2$, there is no correlation with $G_2^{(1)}$. Therefore,

$$I(\bar{A}_1;B|X,U=u)_\omega = I\big(G_2^{(u)};B|X,U=u\big)_\omega \tag{A4}$$

for $u \in \{1,2\}$.

Let $U \sim \text{Bernoulli}(\lambda)$, with $\lambda \in [0,1]$. Observe that the convex combinations of the rates satisfy

$$R_\lambda \equiv (1-\lambda)R_1 + \lambda R_2 \leqslant I(X;B|U)_\omega \leqslant I(XU;B)_\omega, \tag{A5}$$

and

$$R'_\lambda \equiv (1-\lambda)R'_1 + \lambda R'_2 \leqslant I(\bar{A}_1;B|XU)_\omega, \tag{A6}$$

with respect to the following states

$$\omega_{XU\bar{A}_1A} = \sum_{(x,u)\in\mathcal{X}\times\mathcal{U}} p_U(u)p_{X|U}(x|u)|x,u\rangle\langle x,u| \otimes (\text{id}\otimes\bar{\mathcal{F}}^{(x,u)}_{\bar{A}_0\to A})(\varphi_{\bar{A}_1\bar{A}_0}), \tag{A7}$$

$$\omega_{XU\bar{A}_1B} = (\text{id}\otimes\mathcal{N}_{A\to B})(\omega_{XU\bar{A}_1A}). \tag{A8}$$

As we substitute the auxiliary variable $\bar{X} \equiv (X,U)$ in (3), we observe that the pair (R_λ, R'_λ) is in the rate region $\mathcal{R}_{\text{EA}*}(\mathcal{N})$ as well. Thereby, the region is a convex set. ■

2. Proof of Corollary 1

The proof for Corollary 1 follows from the convexity property in Lemma 1. It suffices to consider the boundaries of the two regions in (7).

Consider $(R_1, R'_1) = [I(X;B)_\omega, I(G_2;B|X)_\omega]$. Next, we claim that the rate pair $(R_2, R'_2) = [0, I(XG_2;B)_\omega]$ belongs to $\mathcal{R}_{\text{EA}*}(\mathcal{N})$ as well. To see this, set

$$\widetilde{X} \equiv \emptyset,\ \widetilde{A}_1 \equiv (X,G_2),\ \widetilde{A}_0 \equiv A,\ \text{and}$$
$$\varphi_{\widetilde{A}_1\widetilde{A}_0} \equiv \omega_{XG_2A}. \tag{A9}$$

As for the convex combination of (R_1, R'_1) and (R_2, R'_2), we have

$$R_\lambda \equiv (1-\lambda)R_1 + \lambda R_2 = (1-\lambda)I(X;B)_\omega \tag{A10}$$

and

$$\begin{aligned} R'_\lambda &\equiv (1-\lambda)R'_1 + \lambda R'_2 \\ &= (1-\lambda)I(G_2;B|X)_\omega + \lambda I(XG_2;B)_\omega \\ &= I(G_2;B|X)_\omega + \lambda[I(XG_2;B)_\omega - I(G_2;B|X)_\omega] \\ &= I(G_2;B|X)_\omega + \lambda I(X;B)_\omega \end{aligned} \tag{A11}$$

by the chain rule for the quantum mutual information. By Lemma 1, the pair (R_λ, R'_λ) belongs to the region $\mathcal{R}_{\text{EA}*}(\mathcal{N})$, hence the corollary follows from (A10) to (A11) ■.

[1] Z. Li, K. Xue, J. Li, L. Chen, R. Li, Z. Wang, N. Yu, D. S. Wei, Q. Sun, and J. Lu, Entanglement-assisted quantum networks: Mechanics, enabling technologies, challenges, and research directions, IEEE Commun. Surv. Tut. (2023), doi:10.1109/COMST.2023.3294240.

[2] C. Wang and A. Rahman, Quantum-enabled 6G wireless networks: Opportunities and challenges, IEEE Wireless Commun. **29**, 58 (2022).

[3] S. Hao, H. Shi, W. Li, J. H. Shapiro, Q. Zhuang, and Z. Zhang, Entanglement-assisted communication surpassing the ultimate classical capacity, Phys. Rev. Lett. **126**, 250501 (2021).

[4] E. Shchukin, F. Schmidt, and P. van Loock, Waiting time in quantum repeaters with probabilistic entanglement swapping, Phys. Rev. A **100**, 032322 (2019).

[5] U. Pereg, C. Deppe, and H. Boche, Communication with unreliable entanglement assistance, IEEE Trans. Inf. Theory **69**, 4579 (2023).

[6] A. S. Holevo, The capacity of the quantum channel with general signal states, IEEE Trans. Inf. Theory **44**, 269 (1998).

[7] B. Schumacher and M. D. Westmoreland, Sending classical information via noisy quantum channels, Phys. Rev. A **56**, 131 (1997).

[8] J. Körner, The concept of single-letterization in information theory, in *Open Problems in Communication and Computation* (Springer, New York, 1987), pp. 35–36.

[9] M. M. Wilde, *Quantum Information Theory*, 2nd ed. (Cambridge University Press, Cambridge, England, 2017).

[10] A. El Gamal and Y. Kim, *Network Information Theory* (Cambridge University Press, Cambridge, England, 2011).

[11] C. H. Bennett, P. W. Shor, J. A. Smolin, and A. V. Thapliyal, Entanglement-assisted capacity of a quantum channel and the reverse shannon theorem, IEEE Trans. Inf. Theory **48**, 2637 (2002).

[12] C. Shannon, A mathematical theory of communication, Bell Syst. Tech. J **27**, 379 (1948).

[13] M. Horodecki, P. W. Shor, and M. B. Ruskai, Entanglement breaking channels, Rev. Math. Phys. **15**, 629 (2003).

[14] L. Moravčíková and M. Ziman, Entanglement-annihilating and entanglement-breaking channels, J. Phys. A: Math. Theor. **43**, 275306 (2010).

[15] P. W. Shor, Additivity of the classical capacity of entanglement-breaking quantum channels, J. Math. Phys. **43**, 4334 (2002).

[16] A. S. Holevo, Entanglement-breaking channels in infinite dimensions, Probl. Inf. Trans. **44**, 171 (2008).

[17] M. M. Wilde and M. H. Hsieh, Public and private resource trade-offs for a quantum channel, Quantum Inf. Process. **11**, 1465 (2012).

[18] M. M. Wilde, A. Winter, and D. Yang, Strong converse for the classical capacity of entanglement-breaking and hadamard channels via a sandwiched Rényi relative entropy, Commun. Math. Phys. **331**, 593 (2014).

[19] Q. Wang, S. Das, and M. M. Wilde, Hadamard quantum broadcast channels, Quantum Inf. Process. **16**, 248 (2017).

[20] D. Ding and M. M. Wilde, Strong converse for the feedback-assisted classical capacity of entanglement-breaking channels, Probl. Inf. Trans. **54**, 1 (2018).

[21] U. Pereg, Communication over quantum channels with parameter estimation, IEEE Trans. Inf. Theory **68**, 359 (2022).

[22] C. King, Maximal p-norms of entanglement breaking channels, Quantum Inf. Comput. **3**, 186 (2003).

[23] M. F. Sacchi, Entanglement can enhance the distinguishability of entanglement-breaking channels, Phys. Rev. A **72**, 014305 (2005).

[24] J. Ahiable, D. W. Kribs, J. Levick, R. Pereira, and M. Rahaman, Entanglement breaking channels, stochastic matrices, and primitivity, Linear Algebra Appl. **629**, 219 (2021).

[25] D. W. Kribs, J. Levick, R. Pereira, and M. Rahaman, Entanglement breaking rank via complementary channels and multiplicative domains, arXiv:2211.11909.

[26] R. Devendra, G. Sapra, and K. Sumesh, Degradable strongly entanglement breaking maps, arXiv:2304.00309.

[27] K. Matsumoto, T. Shimono, and A. Winter, Remarks on additivity of the holevo channel capacity and of the entanglement of formation, Commun. Math. Phys. **246**, 427 (2004).

[28] A. Grudka and P. Horodecki, Nonadditivity of quantum and classical capacities for entanglement breaking multiple-access channels and the butterfly network, Phys. Rev. A **81**, 060305(R) (2010).

[29] A. Müller-Hermes and S. Singh, Bi-ppt channels are entanglement breaking, arXiv:2204.01685.

[30] T. S. Cubitt, M. B. Ruskai, and G. Smith, The structure of degradable quantum channels, J. Math. Phys. **49**, 102104 (2008).

[31] C. Hirche and F. Leditzky, Bounding quantum capacities via partial orders and complementarity, IEEE Trans. Inf. Theory **69**, 283 (2023).

[32] C. King, The capacity of the quantum depolarizing channel, IEEE Trans. Inf. Theory **49**, 221 (2003).

[33] C. H. Bennett, P. W. Shor, J. A. Smolin, and A. V. Thapliyal, Entanglement-assisted classical capacity of noisy quantum channels, Phys. Rev. Lett. **83**, 3081 (1999).

[34] D. Leung and J. Watrous, On the complementary quantum capacity of the depolarizing channel, Quantum **1**, 28 (2017).

[35] E. Chitambar and G. Gour, Quantum resource theories, Rev. Mod. Phys. **91**, 025001 (2019).

[36] R. Bassoli, H. Boche, C. Deppe, R. Ferrara, F. H. P. Fitzek, G. Janßen, and S. Saeedinaeeni, *Quantum Communication Networks* (Springer, New York, 2021), Vol. 23.

[37] J. Nötzel and S. DiAdamo, Entanglement-enhanced communication networks, in *Proceedings of IEEE Conference on Quantum Computing and Engineering (QCE'2020)* (IEEE, New York, 2020), pp. 242–248.

[38] J. Nötzel and S. DiAdamo, Entanglement-enabled communication for the internet of things, in *Proceedings of the International Conference on Computer, Information and Telecommunication Systems (CITS'2020)* (IEEE, New York, 2020), pp. 1–6.

[39] J. Nötzel and S. DiAdamo, Entanglement-assisted data transmission as an enabling technology: A link-layer perspective, in *Proceedings of the IEEE International Symposium of Information Theory (ISIT'2020)* (IEEE, New York, 2020), pp. 1955–1960.

[40] J. Nötzel, Entanglement-enabled communication, IEEE J. Sel. Areas Inf. Theory **1**, 401 (2020).

[41] C. H. Bennett and S. J. Wiesner, Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states, Phys. Rev. Lett. **69**, 2881 (1992).

[42] S. Arimoto, An algorithm for computing the capacity of arbitrary discrete memoryless channels, IEEE Trans. Inf. Theory **18**, 14 (1972).

[43] R. Blahut, Computation of channel capacity and rate-distortion functions, IEEE Trans. Inf. Theory **18**, 460 (1972).

[44] R. Ahlswede, The capacity region of a channel with two senders and two receivers, Ann. Probab. **2**, 805 (1974).

[45] M. Wiese and H. Boche, The arbitrarily varying multiple-access channel with conferencing encoders, IEEE Trans. Inf. Theory **59**, 1405 (2013).

[46] Y. Mao, O. Dizdar, B. Clerckx, R. Schober, P. Popovski, and H. V. Poor, Rate-splitting multiple access: Fundamentals, survey, and future research trends, IEEE Commun. Surv. Tutorials **24**, 2073 (2022).
[47] C. E. Shannon, Communication in the presence of noise, Proc. IRE **37**, 10 (1949).
[48] E. Biglieri, J. Proakis, and S. Shamai, Fading channels: Information-theoretic and communications aspects, IEEE Trans. Inf. Theory **44**, 2619 (1998).
[49] W. Ma, L. Zhu, and R. Zhang, Mimo capacity characterization for movable antenna systems, IEEE Trans. Wireless Commun. (2023), doi:10.1109/TWC.2023.3307696.
[50] A. S. Holevo, *Quantum Systems, Channels, Information: A Mathematical Introduction* (Walter de Gruyter, Berlin, 2012), Vol. 16.
[51] U. Pereg, C. Deppe, and H. Boche, Quantum channel state masking, IEEE Trans. Inf. Theory **67**, 2245 (2021).
[52] G. Smith and J. Yard, Quantum communication with zero-capacity channels, Science **321**, 1812 (2008).
[53] U. Pereg, C. Deppe, and H. Boche, The multiple-access channel with entangled transmitters, arXiv:2303.10456.
[54] G. S. Jaeger and A. V. Sergienko, Entanglement sudden death: A threat to advanced quantum key distribution? Nat. Comput. **13**, 459 (2014).
[55] Y. Steinberg, Channels with cooperation links that may be absent, in *Proceedings of the IEEE International Symposium of Information Theory (ISIT'2014)* (IEEE, New York, 2014), pp. 1947–1951.
[56] W. Huleihel and Y. Steinberg, Channels with cooperation links that may be absent, IEEE Trans. Inf. Theory **63**, 5886 (2017).
[57] D. Itzhak and Y. Steinberg, The broadcast channel with degraded message sets and unreliable conference, in *Proceedings of the IEEE International Symposium on Information Theory (ISIT'2017)* (IEEE, New York, 2017), pp. 1043–1047.
[58] D. Itzhak and Y. Steinberg, The broadcast channel with degraded message sets and unreliable conference, IEEE Trans. Inf. Theory **67**, 5623 (2021).
[59] U. Pereg and Y. Steinberg, Arbitrarily varying broadcast channel with uncertain cooperation, in *Proceedings of the International Zurich Seminar on Information and Communication (IZS'2020)* (ETH, Zurich, 2020), pp. 63–67.
[60] L. H. Ozarow, S. Shamai, and A. D. Wyner, Information theoretic considerations for cellular mobile radio, IEEE Trans. Veh. Technol. **43**, 359 (1994).
[61] R. Karasik, O. Simeone, and S. Shamai, Robust uplink communications over fading channels with variable backhaul connectivity, IEEE Trans. Wireless Commun. **12**, 5788 (2013).
[62] Q. Zhuang, E. Y. Zhu, and P. W. Shor, Additive classical capacity of quantum channels assisted by noisy entanglement, Phys. Rev. Lett. **118**, 200503 (2017).
[63] J. G. Proakis, *Digital Communications*, 4th ed. (McGraw-Hill, New York, 2001).