Communication Over Entanglement-Breaking Channels With Unreliable Entanglement Assistance

Uzi Pereg
ECE, Technion

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Motivation

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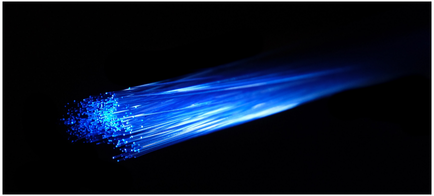
Quantum information technology will potentially boost future 6G systems from both communication and computing perspectives.

Progress in practice:

- Quantum key distribution for secure communication (511 km in optical fibers, 1200 km through space)
  - commercially available: MagiQ, IDQuantique (82k$)
  - development: Toshiba, Airbus EuroQCI



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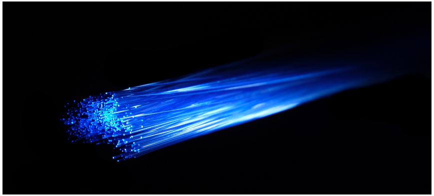

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Motivation: Entanglement Breaking

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- Entanglement breaking is a fundamental property of a large class of quantum channels, mapping any entangled state to a separable state.
  - Examples: classical channels, measurement channels,...
  - qubit depolarizing channels, parameter $\geq 2/3$
- Entanglement-breaking channels are much better understood, compared to general quantum channels
  - single-letter formula [Shor, 2002]
  - strong converse

Motivation: Entanglement

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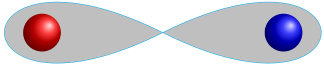
Entanglement resources are instrumental in a wide variety of quantum network frameworks:

- Physical-layer security (device-independent QKD, quantum repeaters) [Vazirani and Vidick 2014] [Yin et al. 2020][Pompili et al. 2021]
- Sensor networks [Xia et al. 2021]
- Communication rate [Bennett et al. 1999] [Hao et al. 2021]
- . . .

Unfortunately, entanglement is a fragile resource that is quickly degraded by decoherence effects.



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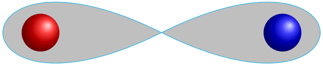

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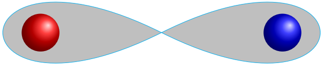

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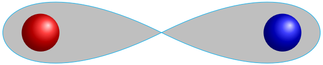
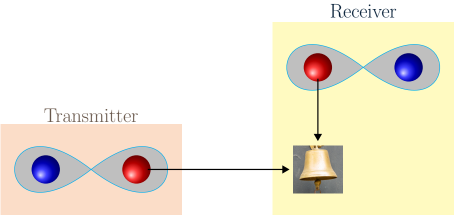


Motivation: Entanglement (Cont.)

- In order to generate (heralded) entanglement in an optical communication system, the transmitter may prepare an entangled pair of photons locally, and then send one of them to the receiver.
- Such generation protocols are not always successful, as photons are easily absorbed before reaching the destination.



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Motivation: Entanglement (Cont.)

- Therefore, practical systems require a back channel. In the case of failure, the protocol is to be repeated. The backward transmission may result in a delay, which in turn leads to a further degradation of the entanglement resources.
- In our previous work, we proposed a new principle of operation: The communication system operates on a rate that is adapted to the status of entanglement assistance. Hence, feedback and repetition are not required. [Pereg et al. 2023]

Unreliable Resources

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Reliability (very partial list):

- Unreliable channel
  - outage capacity [Ozarow, Shamai, and Wyner 1994]
  - automatic repeat request (ARQ) [Caire and Tuninetti 2001] [Steiner and Shamai 2008]
  - cognitive radio [Goldsmith et al. 2008]
  - Network connectivity [Simeone et al. 2012] [Sengupta and Tandon 2015]
- Unreliable cooperation [Steinberg 2014]
  - cribbing encoders [Huleihel and Steinberg 2016]
  - conferencing decoders [Huleihel and Steinberg 2017] [Itzhak and Steinberg 2017] [Pereg and Steinberg 2020]

The Fundamental Problem

Fundamental Problem: Noiseless Channel

Classical Bit-Pipe

The capacity of a classical noiseless bit channel is

$$1 \ \frac{\text{classical bit}}{\text{transmission}}$$

Holevo Bound

The classical capacity of a noiseless qubit channel is

$$1 \ \frac{\text{classical bit}}{\text{transmission}}$$

Fundamental Problem:
Noiseless Channel + Assistance

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Theorem

The classical *common-randomness* (CR) capacity of a noiseless bit-pipe is

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The classical *common-randomness* (CR) capacity of a noiseless bit-pipe is

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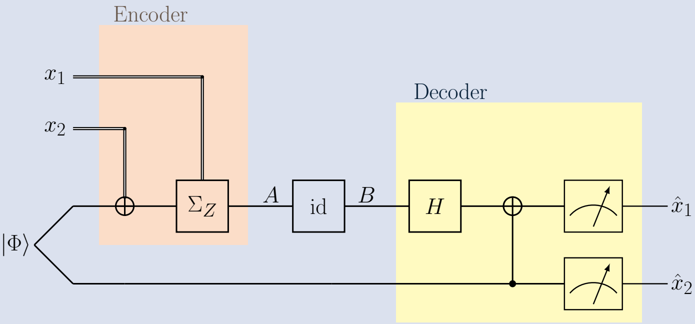
Theorem

The classical *entanglement-assisted* (EA) capacity of a noiseless qubit channel is

$$2 \ \frac{\text{classical bits}}{\textit{transmission}}$$

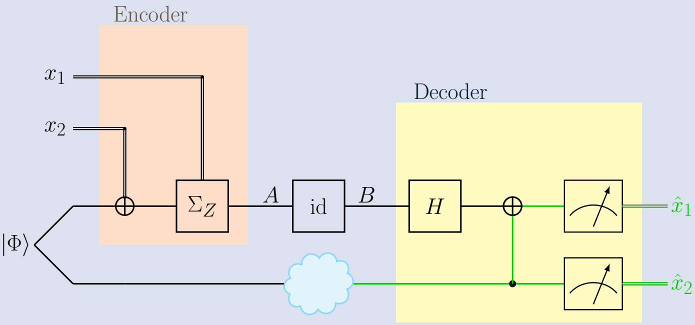
Fundamental Problem:
Noiseless Channel + EA

Superdense Coding



Fundamental Problem:
Noiseless Channel + EA (Cont.)

Superdense Coding



Fundamental Problem: Noiseless Channel + EA (Cont.)

We consider transmission with unreliable EA:
The entangled resource may fail to reach Bob.

Extreme Strategies

1) Uncoded communication
   - Guaranteed rate: $R = 1$
   - Excess rate: $R' = 0$

2) Alice: Employ superdense encoder.

   Bob: If EA is present, employ superdense decoder.
   If EA is absent, abort.

   - Guaranteed rate: $R = 0$
   - Excess rate: $R' = 2$

Fundamental Problem: Noiseless Channel $+$ EA (Cont.)

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Fundamental Problem: Noiseless Channel + EA (Cont.)

Time Division

- Guaranteed rate: $R = 1 - \lambda$
- Excess rate: $R' = 2\lambda$

⋆ Is this optimal?

Main Contributions

- Full capacity characterization for entanglement-breaking channels
- Closed-form capacity formula for the depolarizing channel
- Time division is suboptimal.
  - ⋆ From a networking perspective, this finding is nontrivial and highlights a quantum behavior arising from superposition.

Illustration

Metaphor: N travelers are embarking on a long journey on a ship. Overall, the lifeboats on the ship can accommodate L travelers, $0 \leq L \leq N$. In the event that the ship sinks, $(N - L)$ travelers will be rescued and brought back to their starting point, and the journey will continue with the remaining travelers in the lifeboats.

Port of Haifa, Israel.

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# life boats: L	many	few
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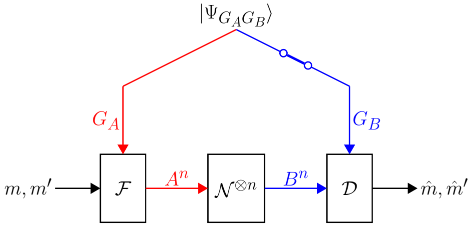
- Division plan: Divide the passengers between a light ship and a heavy ship $\Rightarrow$ $(R, R') = (1 - \lambda)(R_{\text{light}}, R'_{\text{light}}) + \lambda(R_{\text{heavy}}, R'_{\text{heavy}})$.
- Figuratively, our results show that if the journey is subject to a quantum evolution, then we may outperform the division plan by allowing travelers to be in a quantum superposition state between a heavy ship and a light ship.

Coding with Unreliable Assistance

Communication Scheme (1)

Alice chooses two messages, m and m'.

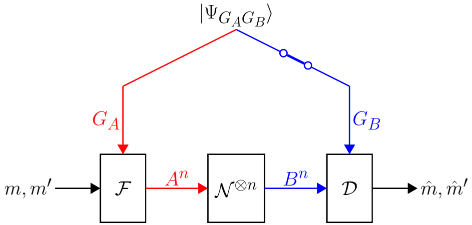


Coding with Unreliable Assistance

Communication Scheme (2)

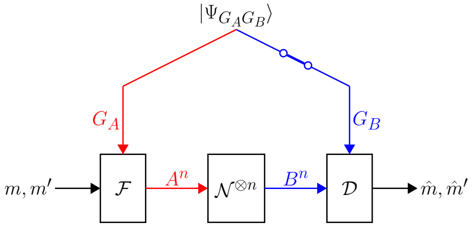
Input: Alice prepares $\rho^{m,m'}_{A^n} = \mathcal{F}^{m,m'}(\Psi_{G_A})$, and transmits A^n.

Output: Bob receives B^n.



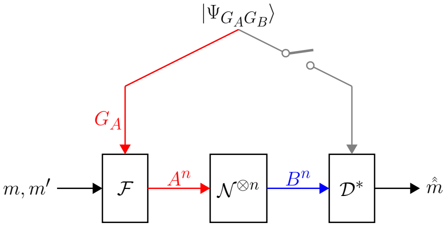
Decoding with Entanglement Assistance

If EA is *present*, Bob performs a measurement $\mathcal{D}$ to estimate m, m'.



Decoding without Assistance

If EA is absent, Bob performs a measurement $\mathcal{D}^*$ to estimate m alone.



Coding with Unreliable Assistance (Cont.)

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Capacity Region

- (R, R') is achievable with unreliable entanglement assistance if there exists a sequence of $(2^{nR}, 2^{nR'}, n)$ codes such that the error probabilities (with and without assistance) tend to zero as $n \to \infty$.
- The capacity region $\mathcal{C}_{\text{EA}*}(\mathcal{N})$ is the closure of the set of achievable rate pairs.

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Entanglement-Breaking Channels

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Definition

A quantum channel $\mathcal{N}_{A\to B}$ is called **entanglement breaking** if for every input state $\rho_{AA'}$, where A' is an arbitrary reference system, the channel output $\mathcal{N}_{A\to B}(\phi_{AE})$ is separable, i.e.,

$$\mathcal{N}_{A\to B}(\phi_{AE}) = \sum_{y\in\mathcal{Y}} p_Y(y)\psi_B^y \otimes \psi_E^y$$

Main Result

Let $\mathcal{N}_{A\to B}$ be a quantum channel. Define

$$\mathcal{R}_{\mathrm{EA}^*}(\mathcal{N}) = \bigcup_{p_X\,,\ \varphi_{G_1 G_2}\,,\ \mathcal{F}^{(x)}} \left\{ \begin{array}{rl} (R,R') \,:\, R \leq & I(X;B)_\rho \\ R' \leq & I(G_2;B|X)_\rho \end{array} \right\}$$

where the union is over all auxiliary variables $X \sim p_X$, bipartite states $\varphi_{G_1 G_2}$, and quantum encoding channels $\mathcal{F}^{(x)}_{G_1\to A}$, with

$$\rho_{XG_2A} = \sum_{x\in\mathcal{X}} p_X(x)\, |x\rangle\langle x| \otimes (\mathrm{id} \otimes \mathcal{F}^{(x)}_{G_1\to A})(\varphi_{G_1 G_2})\,,$$

$$\rho_{XG_2B} = (\mathrm{id} \otimes \mathcal{N}_{A\to B})(\rho_{XG_2A})\,.$$

Main Result (Cont.)

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Theorem

The capacity region of an entanglement-breaking quantum channel $\mathcal{N}_{A\to B}$ with unreliable entanglement assistance is given by

$$\mathcal{C}_{\text{EA}^*}(\mathcal{N}) = \mathcal{R}_{\text{EA}^*}(\mathcal{N})$$

U. Pereg, "Communication over entanglement-breaking channels with unreliable entanglement assistance," *Physical Review A*, vol. 108.4, 042616, October 2023.

Main Result (Cont.)

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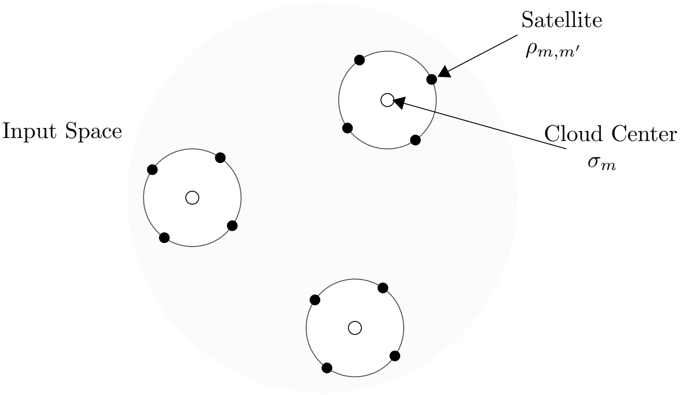
- Single-letterization is highly valued in Shannon theory
  - ✓ computability [Körner, 1987]
  - ✓ uniqueness [Wilde, 2017]
  - ✓ insights on optimal coding [El Gamal and Kim, 2011]

Achievability [Pereg et al., 2023]

— based on a quantum version of “Superposition Coding":



Converse Part

— the main contribution.

Proof is based on the technique from [Pereg, 2022] and geometric properties.

Convexity Properties

Similar properties as for the broadcast channel with degraded message sets:

Lemma 1

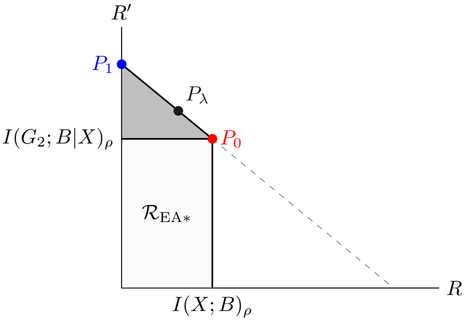
$\mathcal{R}_{\text{EA}^*}$ is a convex set.

Corollary

$$\mathcal{R}_{\text{EA}^*} \supseteq \left\{ \begin{array}{ll} (R, R') \, : \, R \le & (1-\lambda) I(X;B)_\rho \\ R' \le & I(G_2;B|X)_\rho + \lambda I(X;B)_\rho \end{array} \right\}$$

Lemma 2

$$\mathcal{R}_{\text{EA}^*} = \bigcup \left\{ \begin{array}{ll} (R, R') \, : \, R \le & I(X;B)_\rho \\ R + R' \le & I(XG_2;B)_\rho \end{array} \right\}$$



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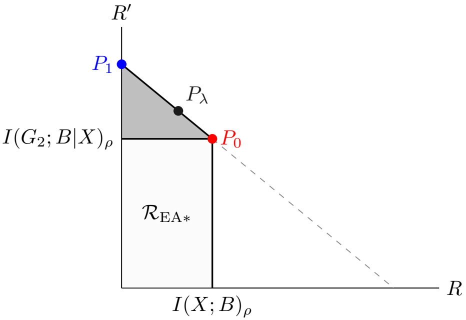

Converse Proof

Every entanglement-breaking can be represented by a measurement channel, followed by a preparation channel: $\mathcal{N}_{A\to B} = \mathcal{P}_{Y\to B} \circ \mathcal{M}_{A\to Y}$. Thus, by DPI,

$$n(R-\varepsilon_n^*) \leq \sum_{i=1}^{n} I(M, B^{i-1}; B_i)_\omega \leq \sum_{i=1}^{n} I(M, Y^{i-1}; B_i)_\omega \equiv \sum_{i=1}^{n} I(X_i; B_i)_\omega \,,$$

and similarly,

$$n(R+R'-\varepsilon_n) \leq \sum_{i=1}^{n} I(M', G_B^{(n)}, X_i; B_i)_\omega \,.$$

Then, introduce a time-sharing variable $\sim \mathrm{Unif}[n]$, as usual.

Single Letterization

Denote the input dimension by $d_A \equiv \dim(\mathcal{H}_A)$.

Lemma

The union is exhausted by pure states $|\phi_{G_1 G_2}\rangle$, cardinality $|\mathcal{X}| \leq d_A^2 + 1$, and dimensions $\dim(\mathcal{H}_{G_1}) = \dim(\mathcal{H}_{G_2}) \leq d_A(d_A^2 + 1)$.

- The first part has already been stated in [Pereg et al., 2023].
- The quantum dimension bound is new.
  Using the mirror lemma, the encoding on G_1 is reflected onto G_2.

Example: Depolarizing Channel

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Qubit depolarizing channel

$$\mathcal{N}(\rho) = (1-\varepsilon)\rho + \varepsilon\frac{\mathbb{1}}{2} \quad , \quad \varepsilon \in \left[\frac{2}{3}, 1\right]$$

Capacity Formula with Unreliable Entanglement Assistance

$$\mathcal{C}_{\text{EA}^*}(\mathcal{N}) = \bigcup_{0\leq\alpha\leq\frac{1}{2}} \left\{ \begin{array}{rl} (R, R') \,:\, R & \leq 1 - h_2\left(\alpha * \frac{\varepsilon}{2}\right) \\ R' & \leq h_2(\alpha) + h_2\left(\alpha * \frac{\varepsilon}{2}\right) - H\left(\frac{\alpha\varepsilon}{2}, \frac{(1-\alpha)\varepsilon}{2}, \beta_+, \beta_-\right) \end{array} \right\}$$

with $\beta_\pm \equiv \frac{1}{2} - \frac{\varepsilon}{4} \pm \sqrt{\frac{\varepsilon^2}{16} - (1-\alpha)\alpha\varepsilon(1-\frac{3\varepsilon}{4}) + \frac{1-\varepsilon}{4}}$, where $H(\mathbf{p}) \equiv -\sum_i p_i \log(p_i)$ is the Shannon entropy, $h_2(x) \equiv H(x, 1-x)$, $\alpha * \beta = (1-\alpha)\beta + \alpha(1-\beta)$.

Example: Depolarizing Channel (Cont.)

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Converse Part

follows from our capacity theorem + observations from [Leung and Watrous, 2017].

Achievability: Quantum Superposition State

Set

$$|\phi_{G_1 G_2}\rangle \equiv \sqrt{1-\alpha}\,|0\rangle \otimes |0\rangle + \sqrt{\alpha}\,|1\rangle \otimes |1\rangle$$

and

$$p_X = \left(\frac{1}{2}, \frac{1}{2}\right) \quad , \quad \mathcal{F}^{(x)}(\rho) \equiv \Sigma_X^x \rho \Sigma_X^x$$

Example: Depolarizing Channel (Cont.)

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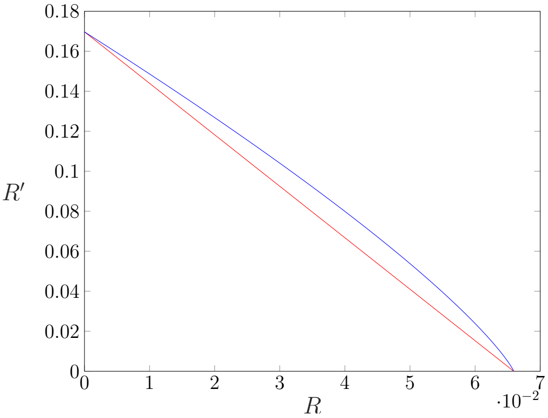
Figure: Capacity region for $\varepsilon = 0.7$.



Summary and Concluding Remarks

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- We considered communication over an entanglement-breaking quantum channel $\mathcal{N}_{A\to B}$, where Alice and Bob are provided with *unreliable* entanglement assistance.

- Our model resembles a broadcast channel $\mathcal{N}_{A\to B_1 B_2}$ when both receivers have the same output[1], yet only one has entanglement assistance.
  - In the classical case: $B_1 \equiv B_2 \Rightarrow$ time division is optimal
  - Surprisingly, in the quantum case, time division is suboptimal.

- Our optimal scheme combines quantum superposition states $+$ superposition coding. Thereby, our findings highlight a quantum behavior arising from superposition.

[1] While this is intuitive, it is physically impossible by the no-cloning theorem.

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Summary and Concluding Remarks (Cont.)

- Security: Eve steals resource [Lederman and Pereg, 2024] `arXiv:2401.12861` **[quant-ph]**

Thank you