The Quantum MAC with Cribbing Encoders

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Motivation: Uplink Communication

- The multiple-access channel (MAC) is among the most fundamental models in network communication and information theory.
  - cellular communication: uplink from mobiles to the base station
  - sat-based IoT: from ground devices to a satellite in space
  - WLAN: from terminals to access point

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Motivation: Uplink Communication (Cont.)

- Signals of different transmitters may interfere with one another.
- In sequential decoding, the receiver decodes the first message while treating the other signals as noise. Subsequently, the previous estimation can reduce the effective noise for decoding the next message.
- If a cognitive transmitter has access to the signal of another transmitter, this knowledge can be exploited such that the receiver will decode the messages with less noise.

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Cooperation in quantum communication networks has been extensively studied in recent years, following both experimental progress and theoretical discoveries.

- Conferencing
  - transmitters [Boche and Nötzel, 2014]
  - receivers [P. et al., 2021]
- Entanglement resources in networks
  - transmitter-receiver [Bennett et al., 1999]
    * Unreliable assistance: "Quantum V" (Session F.12)
  - between transmitters [Leditzky et al., 2020]
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Background: Classical Cribbing

Cribbing is another form of cooperation, whereby one transmitter has access to the other's transmissions.
(very partial list)

- Perfect cribbing [Willems and van der Meulen, 1985]
  - strictly causal; causal; noncausal
- Discretized cribbing [Asnani and Permuter, 2013] [Kopetz et al., 2016]

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- Channel state [Bross and Lapidoth, 2010] [Zamanighomi et al., 2011]
- Unreliable cribbing [Steinberg, 2014] [Huleihel and Steinberg, 2017]
- Cribbing with secrecy [Helal, Bloch, and Nosratinia, 2020]
- ...

Main Contributions

We consider the quantum MAC with cribbing encoders: Transmitter 2 measures a system that is entangled with Transmitter 1.

- Model
  - no cloning: perfect cribbing violates laws of Quantum Mechanics

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  - no cloning: perfect cribbing violates laws of Quantum Mechanics
- Achievable regions
  - strictly causal
  - causal and non-causal: Cribbing inflicts state collapse

Main Contributions (Cont.)

- Robust cribbing
  - multi letter capacity formula

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  - multi letter capacity formula
  - single letter in special cases
- Non-robust cribbing $\rightarrow$ similar to relay
  - partial decode-forward and cutset bounds
  - determine capacity in special cases

Quantum MAC

A quantum multiple-access channel (MAC) $\mathcal{M}_{A_1 A_2 \to B}$ is a linear, completely positive, trace preserving map corresponding to a quantum physical evolution:

$$\rho_{A_1 A_2} \xrightarrow{\mathcal{M}} \rho_B$$

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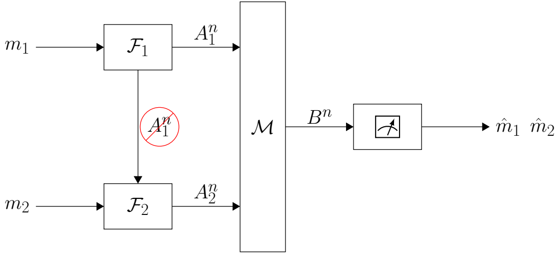
Perfect Cribbing

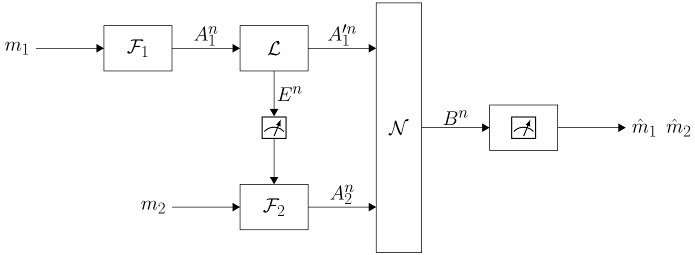
$\Rightarrow$ violatation of the laws of quantum mechanics, by the no-cloning theorem.



Cribbing Encoder

Assume w.l.o.g. that the quantum MAC can be decomposed as

$$\mathcal{M}_{A_1 A_2 \to B}(\rho_{A_1 A_2}) = \mathrm{Tr}_E \left[(\mathcal{N}_{A_1' A_2 \to B} \circ \mathcal{L}_{A_1 \to A_1' E})(\rho_{A_1 A_2}) \right]$$



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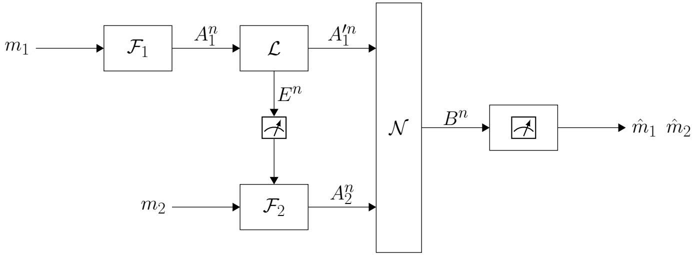
Noisy Cribbing

Alice 1 transmits A_1^n through the cribbing channel $\mathcal{L}_{A_1 \to A_1' E}^{\otimes n}$.



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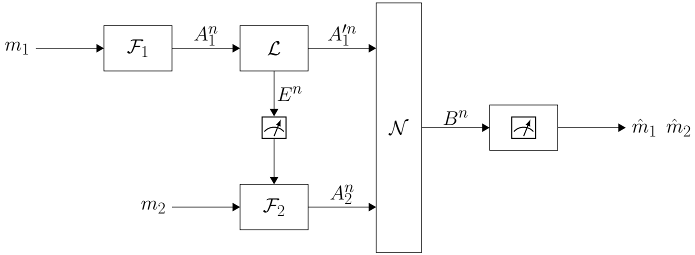
Alice 2 gains access to the system E^n, and performs a measurement.



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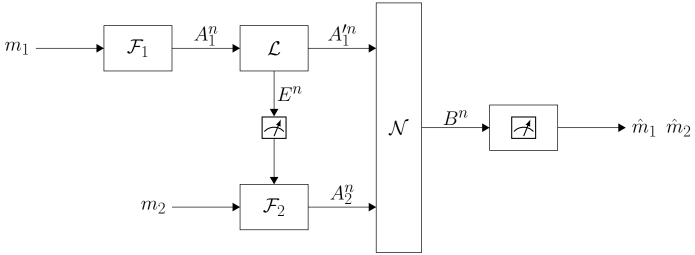
Alice 2 gains access to the system E^n, and performs a measurement.

Then, Alice 2 encodes and transmits A_2^n to Bob.



Cribbing Encoder (Cont.)

The noisy cribbing setting is significantly more challenging, and it is closely related to the relay channel.

It is useful to consider special classes of channels.

Robust Cribbing

Quantum Markov Chain

The quantum systems $A \multimap B \multimap C$ form a Markov chain if $\exists\, \mathcal{P}_{B\to BC}$ such that

$$\rho_{ABC} = (\mathrm{id}_A \otimes \mathcal{P}_{B\to BC})(\rho_{AB})$$

In general, this holds if and only if $I(A;C|B)_\rho = 0$.

Robust Cribbing

Consider an input state $\theta_{A_1 G}$, with G as reference. Let

$$\rho_{A_1' EG} = \mathcal{L}_{A_1 \to A_1' E}(\theta_{A_1 G})$$

We call the cribbing robust if $G \multimap E \multimap A_1'$ form a quantum Markov chain $\forall \theta_{A_1 G}$.

Robust Cribbing (Cont.)

- Intuitively, with robust cribbing, Alice 2's copy is at least as good as the one transmitted through the communication channel.
  That is, A_1' does not contain more information than E.
- Trivial examples:
  - A_1' does not have any information (e.g., $\dim(A_1') = 1$)
  - classical-quantum MAC $\mathcal{M}_{X_1 X_2 \to B}$ with perfect cribbing:

$$E \equiv A_1' \equiv X_1$$

Causality

Strictly Causal

Alice 2 transmits $A_{2,i}$ at time i, and then measures E_i after the transmission. Hence, she only knows the past measurement outcomes z^{i-1} at time i.

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Alice 2 measures the cribbing system E_i at time i, before she transmits. Hence, she knows the past and present measurement outcomes $z^i = (z^{i-1}, z_i)$ at time i.

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Denote capacity regions by $\mathcal{C}_{\text{s-c}}(\mathscr{N} \circ \mathcal{L})$, $\mathcal{C}_{\text{caus}}(\mathscr{N} \circ \mathcal{L})$, and $\mathcal{C}_{\text{n-c}}(\mathscr{N} \circ \mathcal{L})$, resp.

Define

$$\mathcal{R}^{\text{DF}}_{\text{s-c}}(\mathscr{N}\circ\mathcal{L}) = \bigcup_{p_U p_{X_1|U} p_{X_2|U}\,,\;\theta^{x_1}_{A_1}\otimes\zeta^{x_2}_{A_2}} \left\{ \begin{array}{rl} (R_1,R_2)\,:\; R_1 & \leq I(X_1;E|U)_\rho \\ R_2 & \leq I(X_2;B|X_1,U)_\rho \\ R_1+R_2 & \leq I(X_1X_2;B)_\rho \end{array} \right\}$$

where the union is over the distributions of the auxiliary variables U, X_1, X_2, and the ensembles of product input states $\{\theta^{x_1}_{A_1}\otimes\zeta^{x_2}_{A_2}\}$, which yields

$$\begin{aligned} \rho^{u,x_1,x_2}_{A_1'EA_2} &= \mathcal{L}_{A_1\to A_1'E}(\theta^{x_1}_{A_1})\otimes\zeta^{x_2}_{A_2}\,, \\ \rho^{u,x_1,x_2}_{B} &= \mathscr{N}_{A_1'A_2\to B}(\rho^{u,x_1,x_2}_{A_1'EA_2}) \end{aligned}$$

with $|\mathcal{U}|\leq|\mathcal{H}_B|^2+2$, $|\mathcal{X}_1|\leq(|\mathcal{H}_{A_1}|^2+2)(|\mathcal{H}_B|^4+2)$, and $|\mathcal{X}_2|\leq(|\mathcal{H}_{A_2}|^2+1)(|\mathcal{H}_B|^2+2)$.

Results: Strictly-Causal Cribbing (Cont.)

Theorem

Consider a quantum MAC with strictly-causal cribbing.

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$$\mathcal{C}_{s\text{-}c}(\mathscr{N} \circ \mathcal{L}) \supseteq \mathcal{R}^{DF}_{s\text{-}c}(\mathscr{N} \circ \mathcal{L}) .$$

2. *Given robust cribbing,*

$$\mathcal{C}_{s\text{-}c}(\mathscr{N} \circ \mathcal{L}) = \bigcup_{n=1}^{\infty} \frac{1}{n} \mathcal{R}^{DF}_{s\text{-}c}(\mathscr{N}^{\otimes n} \circ \mathcal{L}^{\otimes n}) .$$

⋆ a new result for classical MAC as well

Corollary

The capacity region of the classical-quantum MAC with perfect strictly-causal cribbing is given by

$$\mathcal{C}_{\text{s-c}}(\mathscr{N} \circ \text{id}) = \bigcup_{p_U p_{X_1|U} p_{X_2|U}} \left\{ \begin{array}{rl} (R_1, R_2) \, : \, R_1 & \leq H(X_1|U) \\ R_2 & \leq I(X_2; B|X_1, U)_\omega \\ R_1 + R_2 & \leq I(X_1 X_2; B)_\omega \end{array} \right\}$$

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For the analysis, we derived a generalized quantum packing lemma.

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Decode-Forward Coding Scheme

- Block Markov coding scheme: (in analogy to the classical case)
  - Alice 1 encodes two messages in each block, 'fresh information' & 'resolution'.
  - Alice 2 measures the previous block, decodes the resolution, and then encodes together with her own message.
  - Bob decodes in reversed order (backward decoding).

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- Poor performance if cribbing is too noisy.

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Thereby, Alice 2's measurement may cause a "state collapse" of Alice 1's input.

The cribbing affects both inputs!

Results: Causal and Non-Causal Cribbing (Cont.)

Define

$$\mathcal{R}^{\text{DF}}_{\text{caus}}(\mathscr{N} \circ \mathcal{L}) = \bigcup_{\substack{p_U p_{X_1|U}\,,\ \mathcal{W}_{E\to \bar{E}Z}\,, \\ p_{X_2|Z,U}\,,\ \theta^{x_1}_{A_1} \otimes \zeta^{x_2}_{A_2}}} \left\{ \begin{array}{rl} (R_1, R_2) \,:\, R_1 & \leq I(X_1; \bar{E}Z|U)_\omega \\ R_2 & \leq I(X_2; B|X_1 U)_\omega \\ R_1 + R_2 & \leq I(X_1 X_2; B)_\omega \end{array} \right\}$$

where the union is over the probability distributions $p_U p_{X_1|U}$, the measurement instruments $\mathcal{W}_{E\to\bar{E}Z}(\rho)$, the conditional distributions $p_{X_2|Z,U}$, and the input state collections $\{\theta^{x_1}_{A_1} \otimes \zeta^{x_2}_{A_2}\}$, with

$$\omega_{UX_1A_1'\bar{E}ZX_2A_2} = \sum_{u,x_1,z} p_U(u) p_{X_1|U}(x_1|u) |u\rangle\langle u| \otimes |x_1\rangle\langle x_1|$$

$$\otimes\, W_z \left(\mathcal{L}_{A_1\to A_1'E}(\theta^{x_1}_{A_1})\right) W_z^\dagger \otimes |z\rangle\langle z| \otimes \left(\sum_{x_2} p_{X_2|Z,U}(x_2|z,u) |x_2\rangle\langle x_2| \otimes \zeta^{x_2}_{A_2} \right),$$

$$\omega_{UX_1X_2B} = \mathscr{N}_{A_1'A_2\to B}(\omega_{UX_1X_2A_1'A_2}).$$

Results: Causal and Non-Causal Cribbing (Cont.)

In the last formula, we have a quantum instrument of a measurement,

$$\mathcal{W}_{E\to\bar{E}Z}(\rho)=\sum_z W_z\rho W_z^\dagger\otimes|z\rangle\langle z|$$

where $\bar{E}$ is the post-measurement cribbing system, and Z is the measurement outcome.

Theorem

Consider a quantum MAC with causal cribbing.

1. *The rate region* $\mathcal{R}^{DF}_{caus}(\mathscr{N} \circ \mathcal{L})$ *is achievable, i.e.*

$$\mathcal{C}_{caus}(\mathscr{N} \circ \mathcal{L}) \supseteq \mathcal{R}^{DF}_{caus}(\mathscr{N} \circ \mathcal{L}) \,.$$

2. *For the classical-quantum MAC with perfect cribbing,*

$$\mathcal{C}_{caus}(\mathscr{N} \circ id) = \mathcal{C}_{n\text{-}c}(\mathscr{N} \circ id) = \bigcup_{p_{X_1 X_2}} \left\{ \begin{array}{rl} (R_1, R_2) : \; R_1 & \leq H(X_1) \\ R_2 & \leq I(X_2; B|X_1)_\omega \\ R_1 + R_2 & \leq I(X_1 X_2; B)_\omega \end{array} \right\}$$

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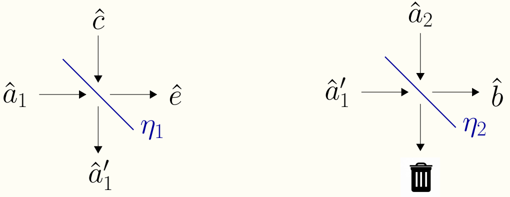
- ⋆ Part 1 is a new result for the classical MAC as well
- ⋆ Part 2 extends the classical result of Willems and van der Meullen (1985) to the classical-quantum MAC with perfect cribbing.

Bosonic MAC

Bosonic MAC with Noisy Cribbing

Consider a single-mode bosonic MAC that is composed of beam splitters as illustrated below:



Bosonic MAC (Cont.)

With strictly-causal cribbing, we obtain

$$\mathcal{C}_{\text{s-c}}(\mathscr{N} \circ \mathcal{L}) \supseteq$$

$$\left\{ \begin{array}{rl} (R_1, R_2) \,:\, R_1 & \leq g(\eta_1 N_{A_1} + (1-\eta_1)N_C) - g((1-\eta_1)N_C) \\ R_2 & \leq g(\eta_2\eta_1 N_C + (1-\eta_2)N_{A_2}) - g(\eta_2\eta_1 N_C) \\ R_1 + R_2 & \leq g(\eta_2(1-\eta_1)N_{A_1} + \eta_2\eta_1 N_C + (1-\eta_2)N_{A_2}) - g(\eta_2\eta_1 N_C) \end{array} \right\}$$

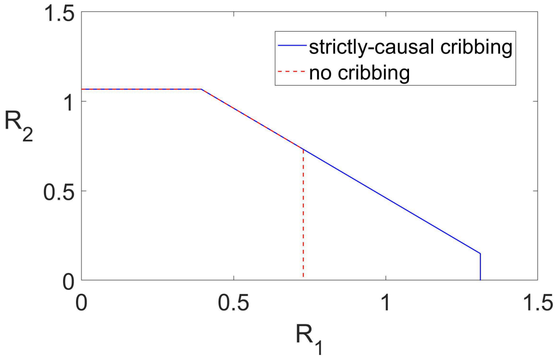
where $g(N) = (N+1)\log(N+1) - N\log(N)$. On the other hand, without cribbing, [Yen and Shapiro, 2005]

$$\mathcal{C}_{\text{none}}(\mathscr{N} \circ \mathcal{L}) =$$

$$\left\{ \begin{array}{rl} (R_1, R_2) \,:\, R_1 & \leq g(\eta_2(1-\eta_1)N_{A_1} + \eta_2\eta_1 N_C) - g(\eta_2\eta_1 N_C) \\ R_2 & \leq g(\eta_2\eta_1 N_C + (1-\eta_2)N_{A_2}) - g(\eta_2\eta_1 N_C) \\ R_1 + R_2 & \leq g(\eta_2(1-\eta_1)N_{A_1} + \eta_2\eta_1 N_C + (1-\eta_2)N_{A_2}) - g(\eta_2\eta_1 N_C) \end{array} \right\}$$

Bosonic MAC (Cont.)

The decode-forward achievable region with strictly-causal cribbing and the capacity region without cribbing are depicted in the figure below:



Define

$$\mathcal{R}^{\text{PDF}}_{\text{s-c}}(\mathscr{N}\circ\mathcal{L}) = \bigcup_{p_{UV}p_{X_1|U,V}p_{X_2|U,V}\,,\ \theta^{x_1}_{A_1}\otimes\zeta^{x_2}_{A_2}} \left\{ \begin{array}{rl} (R_1,R_2)\ :\ R_1 & \le I(V;E|U)_\omega + I(X_1;B|X_2UV)_\omega \\ R_2 & \le I(X_2;B|X_1U)_\omega \\ R_1+R_2 & \le I(X_1X_2;B)_\omega \end{array} \right\}$$

with $\omega^{u,v,x_1,x_2}_{A_1'EA_2} = \mathcal{L}_{A_1\to A_1'E}(\theta^{x_1}_{A_1})\otimes\zeta^{x_2}_{A_2}$ and $\omega^{u,v,x_1,x_2}_{B} = \mathscr{N}_{A_1'A_2\to B}(\omega^{u,v,x_1,x_2}_{A_1'A_2})$.

The superscript ‘PDF’ stands for partial decode-forward coding.

Theorem

The rate region $\mathcal{R}^{PDF}_{s\text{-}c}(\mathscr{N}\circ\mathcal{L})$ is achievable for the quantum MAC with strictly-causal cribbing.

Results: Classical-Quantum MAC

For the c-q MAC $\mathscr{N}_{X_1 X_2 \to B} \circ Q_{Z|X_1}$ with *noisy* cribbing, we also prove a cutset outer bound:

$$\mathcal{R}^{\text{CS}}_{\text{s-c}}(\mathscr{N} \circ Q) = \bigcup_{p_U p_{X_1|U} p_{X_2|U}} \left\{ \begin{array}{rl} (R_1, R_2) \ : \ R_1 & \leq I(X_1; BZ|X_2 U)_\omega \\ R_2 & \leq I(X_2; B|X_1 U)_\omega \\ R_1 + R_2 & \leq I(X_1 X_2; B)_\omega \end{array} \right\}$$

Results: Classical-Quantum MAC

Theorem

1. *The capacity region of the c-q MAC with strictly-causal noisy cribbing is bounded by*

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⋆ Part 2 extends the classical result of Asnani and Permuter (2013) on partial cribbing to the classical-quantum MAC.

Conclusion

We considered the quantum MAC with a cribbing encoder.

- The quantum description is more delicate. Perfect cribbing is against the laws of nature (no-cloning theorem).
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